Probability Theory
Unit 1

1. What do you mean by probability?

Probability is a branch of mathematics that measures the likelihood of an event occurring. It is expressed as a number between 0 and 1, where 0 means the event will not occur, and 1 means the event is certain to occur. Probability helps in predicting outcomes in uncertain situations, such as rolling a die, drawing a card, or forecasting the weather. It plays a crucial role in fields like statistics, finance, and science.

Bayes' Theorem describes how to update probabilities based on new evidence. It is given by:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Meaning:

- **P(A | B):** Probability of event A given B (posterior probability).
- **P(B | A):** Probability of B given A (likelihood).
- **P(A):** Probability of A (prior probability).
- **P(B):** Total probability of B.

Definitions:

8. (a) Trial

A trial is an action or experiment that results in one of several possible outcomes, like tossing a coin or rolling a die.

(b) Sure Event

A sure event is an event that always occurs. Its probability is 1. For example, getting a number between 1 and 6 when rolling a fair die.

(c) Impossible Event

An impossible event never occurs, and its probability is 0. For example, rolling a 7 on a standard six-sided die.

(d) Favorable Cases

Favorable cases are the outcomes that satisfy a given event. For example, in rolling a die, favorable cases for getting an even number are 2, 4, and 6.

(e) Sample Space

The sample space is the set of all possible outcomes of an experiment. For example, for rolling a die, the sample space is {1, 2, 3, 4, 5, 6}.

(f) Event

An event is a subset of the sample space representing a particular outcome or group of outcomes. For example, getting an even number when rolling a die is an event {2, 4, 6}.

(g) Random Variable

A random variable is a function that assigns numerical values to the outcomes of a random experiment. For example, in a coin toss, let X = 1 if heads and X = 0 if tails.

(h) Independent Events

Two events are independent if the occurrence of one does not affect the probability of the other. For example, rolling two dice separately.

(i) Dependent Events

Two events are dependent if the occurrence of one affects the probability of the other. For example, drawing two cards from a deck without replacement.

The **classical definition of probability**, introduced by Pierre-Simon Laplace, states that if an experiment has **N** equally likely outcomes and an event **A** can occur in **m** ways, then the probability of **A** is:

$$P(A) = \frac{m}{N}$$

Key Points:

- Assumes all outcomes are equally likely.
- Works well for simple cases like rolling dice or drawing cards.

Compound probability refers to the probability of two or more events happening together. It can involve independent or dependent events and is calculated using specific rules.

Types of Compound Probability:

1. **For Independent Events (Events that do not affect each other):**

$$P(A \cap B) = P(A) \times P(B)$$

   Example: Rolling two dice—getting a 3 on the first die and a 5 on the second.

2. **For Dependent Events (Events that affect each other):**

$$P(A \cap B) = P(A) \times P(B|A)$$

The **axiomatic definition of probability,** introduced by **Andrey Kolmogorov**, is based on three fundamental axioms:

1. **Non-Negativity:** The probability of any event **A** is always **≥ 0** → $P(A) \geq 0$.
2. **Normalization:** The probability of the entire sample space **S** is **1** → $P(S) = 1$.
3. **Additivity:** For any two mutually exclusive events **A** and **B**, the probability of their union is the sum of their probabilities → $P(A \cup B) = P(A) + P(B)$.

These axioms provide a solid mathematical foundation for probability theory, ensuring consistency in calculations.

Conditional probability is the probability of an event occurring given that another event has already occurred. It is denoted as **P(A | B)**, which means "the probability of event A occurring given that event B has occurred."

Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where:

- **P(A | B)** = Probability of A given B
- **P(A ∩ B)** = Probability of both A and B occurring
- **P(B)** = Probability of B (assuming P(B) > 0)

The **frequentist definition of probability** states that the probability of an event is the long-run relative frequency of that event occurring in repeated independent trials under identical conditions.

Mathematical Definition:

If an experiment is repeated **n** times and an event **A** occurs **m** times, then the probability of **A** is given by:

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

This means that as the number of trials increases, the observed relative frequency of the event stabilizes around a fixed value, which is considered the probability.