Regression Analysis

① Determine regression coefficient of x on y, y on x, correlation coefficient between x and y. Also derive regression lines/equations of x on y and y on x. Hence predict the value of x when y = 10 and y when x = 16

ⓐ x: 10 8 5 4 2 1
y: 4 6 5 7 8 9

soln

Working Table

x	y	xy	x^2	y^2
10	4	40	100	16
8	6	48	64	36
5	5	25	25	25
4	7	28	16	49
2	8	16	4	64
1	9	9	1	81

$\Sigma x = 30$ $\Sigma y = 39$ $\Sigma xy = 166$ $\Sigma x^2 = 210$ $\Sigma y^2 = 271$

$n = 6$

Now ⓘ Regression coefficient of x on y (b_{xy})

$$= \frac{n \times \Sigma xy - \Sigma x \times \Sigma y}{n \times \Sigma y^2 - (\Sigma y \times \Sigma y)} = \frac{(6 \times 166) - (30 \times 39)}{6 \times 271 - 39 \times 39} = \frac{996 - 1170}{1626 - 152}$$

$$= \frac{-174}{105} = -1.65$$

ⓘⓘ Regression coefficient of y on x (b_{yx})

$$= \frac{n \times \Sigma xy - \Sigma x \times \Sigma y}{n \times \Sigma x^2 - (\Sigma x \times \Sigma x)} = \frac{(6 \times 166) - (30 \times 39)}{6 \times 210 - 30 \times 30} = \frac{996 - 1170}{1260 - 900} = \frac{-170}{360}$$

$$= -0.4833$$

ⓘⓘⓘ Correlation coefficient between x and y is

$$= -\sqrt{b_{xy} \times b_{yx}} = -\sqrt{1.6571 \times 0.4833} = -0.8949$$

(iv) Regression line of x on y is

$(x-\bar{x}) = b_{xy} \times (y-\bar{y})$

$\Rightarrow x-5 = -1.66(y-6.5)$

$\Rightarrow x = -1.66y + 10.79 + 5$

$\Rightarrow x = 15.79 - 1.66y \longrightarrow (1)$

$\bar{x} = \frac{\Sigma x}{n} = \frac{30}{6} = 5$

$\bar{y} = \frac{\Sigma y}{n} = \frac{39}{6} = 6.5$

Now, when $y=10$

$x = 15.79 - 1.66 \times 10 = 15.79 - 16.60 = -0.81$

(v) Again Regression line of y on x is

$(y-\bar{y}) = b_{yx} \times (x-\bar{x})$

$\Rightarrow y-6.5 = -0.48 \times (x-5)$

$\Rightarrow y = -0.48x + 6.5 + 2.4$

$\Rightarrow y = 8.9 - 0.48x$

And when $x=16$

$y = -0.48 \times 16 + 8.9 = 8.90 - 7.68 = 1.22$

(b) $\Sigma x = 30$, $\Sigma y = 25$, $\Sigma x^2 = 50$, $\Sigma y^2 = 40$, $\Sigma xy = 80$, $n = 20$

Soln

(i) Regression coefficient of x on y is (b_{xy})

$= \frac{n \times \Sigma xy - \Sigma x \times \Sigma y}{n \times \Sigma y^2 - \Sigma y \times \Sigma y} = \frac{20 \times 80 - 30 \times 25}{20 \times 40 - 25 \times 25} = \frac{1600-750}{800-625} = \frac{850}{175} = 4.857$

(ii) Regression coefficient of y on x is (b_{yx})

$= \frac{n \times \Sigma xy - \Sigma x \times \Sigma y}{n \times \Sigma x^2 - \Sigma x \times \Sigma x} = \frac{20 \times 80 - 30 \times 25}{20 \times 50 - 30 \times 30} = \frac{1600-750}{1000-900} = \frac{850}{100} = 8.50$

(iii) Correlation coefficient between x and y is

$= +\sqrt{b_{xy} \times b_{yx}} = +\sqrt{4.857 \times 8.50} = 6.4253$ (impossible)

(iv) Regression line of x on y is

$(x-\bar{x}) = b_{xy} \times (y-\bar{y})$

$\Rightarrow x - 1.50 = 4.86 \times (y - 1.25)$

$\Rightarrow x = 4.86y + 1.50 - 6.075$

$\bar{x} = \frac{\Sigma x}{n} = \frac{30}{20} = 1.5$

$\bar{y} = \frac{\Sigma y}{n} = \frac{25}{20} = 1.25$

$\Rightarrow x = 4.86y - 4.575$

Now, when $y = 10$

$x = 4.86 \times 10 - 4.575 = 48.6 - 4.575 = 44.025$

(v) Regression line of y on x is

$(y - \bar{y}) = b_{yx} \times (x - \bar{x})$

$\Rightarrow y - 1.25 = 8.50 \times (x - 1.50)$

$\Rightarrow y = 8.50x + 1.25 - 12.75$

$\Rightarrow y = 8.50x - 11.50$

when $x = 16$

$y = 8.50 \times 16 - 11.50 = 136 - 11.50 = 124.50$

(2) Given data of a company-

Particulars	Sales (₹)	Advertisement (₹)
Average (Mean)	70	12
Standard Deviation	8	3
Correlation coefficient	0.70	

(i) Estimate the likely advertisement expenditure if the company has a sale target of ₹100.

(ii) Estimate the likely sale if the company has advertisement expenditure of ₹15.

Soln Let x = Sales ; y = advertisement expenditure

Given, $\bar{x} = 70$, $\bar{y} = 12$, $\sigma_x = 8$; $\sigma_y = 3$, $r = 0.70$

Now,

$b_{xy} = r \times \frac{\sigma_x}{\sigma_y} = 0.70 \times 8/3 = 1.87$

$b_{yx} = r \times \frac{\sigma_y}{\sigma_x} = 0.70 \times 3/8 = 0.26$

(i) Regression equation of y on x is

$(y - \bar{y}) = b_{yx} \times (x - \bar{x})$

$\Rightarrow y - 12 = 0.26 \times (x - 70)$

$\Rightarrow y = 0.26x + 12 - 18.20$

$\Rightarrow y = 0.26x - 6.20$

when $x = 100$

$\therefore y = 0.26 \times 100 - 6.20 = 26 - 6.20 = 19.20$

Ans→ The probable expenditure is ₹19.20

(ii) Regression equation of x on y is

$(x-\bar{x}) = b_{xy} \times (y-\bar{y})$

$\Rightarrow x-70 = 1.87 \times (y-12)$

$\Rightarrow x = 1.87y + 70 - 22.44$

$\Rightarrow x = 1.87y + 47.56$

when $y = 15$

$\therefore x = 1.87 \times 15 + 47.56 = 75.61$

Ans $\Rightarrow$ The probable sales is ₹ 75.61

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(3) Given regression equations are

$3x-2y = 24$, $2x-5y = 5$ and variance $(x) = 4$

i) Identify the regression equation of x on y and y on x.

ii) Find variance (y)

iii) Find average or mean values of x and y

iv) Predict x when $y = 16$

v) Predict y when $x = 11$.

soln

(i) Let $3x-2y=24$ be regression equation of x on y and $2x-5y=5$ be regression equation of y on x

Now, $3x-2y=24$

$\Rightarrow 3x = 24+2y$

$\Rightarrow \frac{3x}{3} = \frac{24}{3} + \left(\frac{2}{3}\right)y \longrightarrow$ (I)

$\Rightarrow x = 8 + 0.67y$

$\therefore b_{xy} = 0.67$

Again, $2x-5y=5$

$\Rightarrow 2x-5 = 5y$

$\Rightarrow \left(\frac{2}{5}\right)x - \left(\frac{5}{5}\right) = \frac{5y}{5}$

$\Rightarrow y = 0.4x - 1 \longrightarrow$ (II)

$\therefore b_{yx} = 0.40$

Now, $r = +\sqrt{b_{xy} \times b_{yx}} = \sqrt{0.67 \times 0.40} = 0.5177$ (valid)

$\therefore 3x-2y=24$ is regression equation of x on y

and $2x-5y=5$ " " " " y on x

(ii) Given, Variance $(x) = 4$

$\therefore$ S.D of $(x) = \sqrt{4} = 2$

We know $b_{xy} = r \times \frac{\sigma x}{\sigma y}$

$\Rightarrow 0.67 = \frac{0.5177 \times 2}{\sigma y}$

$\Rightarrow \sigma y = 1.5453$

$\therefore$ Variance $(y) = (1.5453)^2 = 2.3888$

(iii) Given, $3x - 2y = 24 \longrightarrow$ (I)

$2x - 5y = 5 \longrightarrow$ (II)

Taking mean on both sides, we get

$3\bar{x} - 2\bar{y} = 24 \longrightarrow$ (III)

$2\bar{x} - 5\bar{y} = 5 \longrightarrow$ (IV)

Now (III) $\times 2 \Rightarrow 6\bar{x} - 4\bar{y} = 48$

(IV) $\times 3 \Rightarrow 6\bar{x} - 15\bar{y} = 15$

$\quad (-) \quad (+) \quad (-)$

$11\bar{y} = 33$

$\Rightarrow \bar{y} = 3$

From (III)

$3\bar{x} - 2 \times 3 = 24$

$\Rightarrow 3\bar{x} = 24 + 6$

$\Rightarrow \bar{x} = \frac{30}{3}$

$\Rightarrow \bar{x} = 10$

(iv) We have, $x = 0.67y + 8$ and $y = 6$

$\Rightarrow x = 0.67 \times 6 + 8$

$\Rightarrow x = 4 + 8$

$\Rightarrow x = 12$

(v) Also we have $y = 0.4x - 1$ and $x = 11$

$\Rightarrow y = 0.4 \times 11 - 1$

$\Rightarrow y = 4.4 - 1$

$\Rightarrow y = 3.4$..

Correlation

① Determine Karl Pearson correlation coefficient between 2 variables

x: 2 5 3 6 1

y: 4 7 5 6 2

Soln

x	y	xy	x^2	y^2
2	4	8	4	16
5	7	35	25	49
3	5	15	9	25
6	6	36	36	36
1	2	2	1	4
$\Sigma x = 17$	$\Sigma y = 24$	$\Sigma xy = 96$	$\Sigma x^2 = 75$	$\Sigma y^2 = 130$

$\therefore n = 5$

we know, Karl Pearson correlation coefficient,

$$r = \frac{n \times \Sigma xy - \Sigma x \times \Sigma y}{\sqrt{n \times \Sigma x^2 - (\Sigma x \times \Sigma x)} \times \sqrt{n \times \Sigma y^2 - (\Sigma y \times \Sigma y)}}$$

$$= \frac{(5 \times 96) - (17 \times 24)}{\sqrt{5 \times 75 - (17 \times 17)} \times \sqrt{5 \times 130 - (24 \times 24)}}$$

$$= \frac{480 - 408}{\sqrt{375 - 289} \times \sqrt{650 - 576}}$$

$$= \frac{72}{\sqrt{86} \times \sqrt{74}} = \frac{74}{79.77} = 0.9025$$

② Determine Spearman rank correlation coefficient

ⓐ Marks by 1st Judge : 60 68 70 57 59 65

" by 2nd " : 62 70 67 75 80 66

Soln Let x = Marks by 1st Judge

y = " " 2nd Judge.

R_x = Rank in 'x' series.

R_y = " " 'y' " .

x	y	R_x	R_y	$d=R_x-R_y$	d^2
60	62	4	6	-2	4
68	70	2	3	-1	1
70	67	1	4	-3	9
57	75	6	2	4	16
59	80	5	1	4	16
65	66	3	5	-2	4
					$\Sigma d^2=50$

Now Spearman's Rank correlation,

$$r_{xy} = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6\times50}{6(6^2-1)} = 1 - \frac{300}{210} = 1 - 1.42857$$

$$= -0.42857$$

(b) Marks by Judge A : 80 72 66 72 65 72 60 80

" " " B : 90 97 98 67 66 45 97 66

Soln Let x = Marks by Judge A

y = " " " B

R_x = Rank in x series

R_y = " " y series.

x	y	R_x	R_y	$d=R_x-R_y$	d^2
80	90	1.5	4	-2.5	6.25
72	97	4	2.5	1.5	2.25
66	98	6	1	5	25
72	67	4	5	1	1
65	66	7	6.5	0.5	0.25
72	45	4	8	-4	16
60	97	8	2.5	5.5	30.25
80	66	1.5	6.5	-5	25
					$\Sigma d^2=106$

In x series '80' is repeated 2 times

72 is repeated 3 times

In y series 97 is repeated 2 times

66 is repeated 2 times

Now, Spearman's Rank Correlation,

$$r_{xy} = 1 - \frac{6\left[\Sigma d^2 + \Sigma\left(\frac{m^3-m}{12}\right)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[106 + \frac{2^3-2}{12} + \frac{3^3-3}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12}\right]}{8(64-1)}$$

$$= 1 - \frac{6[106 + 0.5 + 2 + 0.5 + 0.5]}{8 \times 63}$$

$$= 1 - \frac{109.5}{84} = \frac{84 - 109.5}{84} = -0.3035$$

(8) A computer while calculating the correlation co-efficient between X and Y obtained the following results:
$\Sigma x = 120$, $\Sigma y = 90$, $\Sigma x^2 = 600$, $\Sigma y^2 = 250$, $\Sigma xy = 356$, $n = 30$
It was however later on discovered that 2 pairs of observations (8,10) and (12,7) were wrong instead of correct pairs (8,12) and (10,8) find the correct correlation coefficient.

Soln Given, $n = 30$; $\Sigma x = 120$; $\Sigma y = 90$; $\Sigma x^2 = 600$; $\Sigma y^2 = 250$; $\Sigma xy = 356$.
Since 2 pairs of observations (8,10) and (12,7) were wrong instead of correct pairs (8,12) and (10,8)

$\therefore$ Correct $\Sigma x = 120 + 8 + 10 - 8 - 12 = 118$

" $\Sigma y = 90 + 12 + 8 - 10 - 7 = 93$

" $\Sigma x^2 = 600 + 8^2 + 10^2 - 8^2 - 12^2 = 556$

" $\Sigma y^2 = 250 + 12^2 + 8^2 - 10^2 - 7^2 = 309$

" $\Sigma xy = 356 + (8 \times 12) + (10 \times 8) - (8 \times 10) - (12 \times 7)$
$= 368$

$\therefore$ correct correlation coefficient is

$$r_{xy} = \frac{(n \times \Sigma xy) - (\Sigma x \times \Sigma y)}{\sqrt{n \times \Sigma x^2 - (\Sigma x \times \Sigma x)} \times \sqrt{n \times \Sigma y^2 - (\Sigma y \times \Sigma y)}}$$

$$= \frac{(30 \times 368) - (118 \times 93)}{\sqrt{30 \times 556 - 118 \times 118} \times \sqrt{30 \times 309 - 93 \times 93}}$$

$$= \frac{11040 - 10974}{\sqrt{2756} \times \sqrt{621}} = \frac{66}{1308.04} = 0.05045$$

Time Series Analysis

① Determine 3 Yearly, 4 Yearly, 5 Yearly, 6 Yearly and 7 Yearly moving averages from the following data

Years: 2000 2001 2002 2003 2004 2005 2006 2007
Sales: 20 22 25 28 30 32 34 36

Soln

3 Yearly Moving Averages

Years	Sales	3 Yearly Moving Total	3 Yearly Moving Avg
2000	20		
2001	22 →	20+22+25=67	67/3 = 22.33
2002	25 →	22+25+28 = 75	75/3 = 25
2003	28 →	25+28+30 = 83	83/3 = 27.67
2004	30 →	28+30+32 = 90	90/3 = 30
2005	32 →	30+32+34 = 96	96/3 = 32
2006	34 →	32+34+36 = 102	102/3 = 34
2007	36		

4 Yearly Moving Averages

Years	Sales	4 Yearly Moving Total	4 Yearly Moving Avg	4 Yearly center Moving Avg
2000	20			
2001	22 →	20+22+25+28 = 95	95/4 = 23.75	
2002	25 →	22+25+28+30 = 105	105/4 = 26.25	$\frac{23.75+26.25}{2} = 25$
2003	28 →	25+28+30+32 = 115	115/4 = 28.75	$\frac{26.25+28.75}{2} = 27.5$
2004	30 →	28+30+32+34 = 124	124/4 = 31	$\frac{28.75+31}{2} = 29.88$
2005	32 →	30+32+34+36 = 132	132/4 = 33	$\frac{31+33}{2} = 32$
2006	34			
2007	36			

5 Yearly Moving Averages

Years	Sales	5 Yearly Moving Total	5 Yearly Moving Avg
2000	20		
2001	22		
2002	25 →	20+22+25+28+30 = 125	125/5 = 25
2003	28 →	22+25+28+30+32 = 137	137/5 = 27.4
2004	30 →	25+28+30+32+34 = 149	149/5 = 29.8
2005	32 →	28+30+32+34+36 = 160	160/5 = 32
2006	34		
2007	36		

6 Yearly Moving Average

Years	Sales	6 Yearly Moving Total	6 Yearly Moving Avg	6 Yearly center Moving Avg
2000	20			
2001	22			
2002	25 →	20+22+25+28+30+32 = 157	157/6 = 26.17	$\frac{26.17+28.5}{2}$ = 27.3
2003	28 →	22+25+28+30+32+34 = 171	171/6 = 28.5	$\frac{28.5+30.83}{2}$ = 29.665
2004	30 →	25+28+30+32+34+36 = 185	185/6 = 30.83	
2005	32			
2006	34			
2007	36			

7 Yearly Moving Average

Years	Sales	7 Yearly Moving Total	7 Yearly Moving Avg
2000	20		
2001	22		
2002	25		
2003	28 →	20+22+25+28+30+32+34 = 191	191/7 = 27.28
2004	30 →	22+25+28+30+32+34+36 = 207	207/7 = 29.57
2005	32		
2006	34		
2007	36		

(2) Using least squares method fit/derive a straight line trend equation from the following data:

(a) Years: 2000 2001 2002 2003 2004
Sales: 40 42 46 50 55

(b) Years: 2000 2001 2002 2003 2004 2005 2006 2007
Sales: 60 62 68 65 70 80 82 90

Soln (a) Let $y = a + bx$ be a straight line trend equation where 'y' denotes sales and 'x' denotes time such that $x = \frac{\text{Given Year} - 2002}{1}$ = Given Year − 2002

The values of a and b can be determined by solving the following two equations:

$\Sigma y = na + b\Sigma x \rightarrow$ (I)

$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow$ (II)

Working Table

Years	Sales(Y)	X = Years - 2002	X^2	XY
2000	40	-2	4	-80
2001	42	-1	1	-42
2002	46	0	0	0
2003	50	1	1	50
2004	55	2	4	110
	$\Sigma Y = 233$	$\Sigma x = 0$	$\Sigma x^2 = 10$	$\Sigma xy = 38$

n = 5

From (I)

$233 = 5a + b \times 0$

$\Rightarrow 233/5 = a$

$\Rightarrow 46.6 = a$

From (II) $38 = 46.6 \times 0 + b \times 10$

$\Rightarrow 38 = 0 + 10b$

$\Rightarrow 38/10 = b$

$\Rightarrow 3.8 = b$

∴ Required straight line trend equation is

$y = a + bx$

$\Rightarrow y = 46.6 + 3.8x$

(b) Soln Let $y = a + bx$ be a straight line trend equation where 'y' denotes sales and 'x' denotes time such that

$x = \frac{\text{Years} - 2003.5}{0.5}$

The values of a and b can be determined by solving the following 2 equations:

$\Sigma y = na + b\Sigma x \rightarrow$ (I)

$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow$ (II)

Working Table

Years	Sales(Y)	X = Years - 2003.5 / 0.5	X^2	XY
2000	60	-7	49	-420
2001	62	-5	25	-310
2002	68	-3	9	-204
2003	65	-1	1	-65

2004	70	1	1	70
2005	80	3	9	240
2006	82	5	25	410
2007	90	7	49	630
	$\Sigma Y = 577$	$\Sigma X = 0$	$\Sigma X^2 = 168$	$\Sigma XY = 351$

$n = 8$

from (I)

$577 = 8 \times a + b \times 0$

$\Rightarrow 577/8 = a$

$\Rightarrow 72.125 = a$

from (II) $351 = a \times 0 + 168 \times b$

$\Rightarrow 351 = 168b$

$\Rightarrow 351/168 = b$

$\Rightarrow 2.0892 = b$

$\therefore$ Required straight line trend equation is

$y = a + bx$

$\Rightarrow y = 72.125 + 2.0892x$

Probability

① 3 coins are tossed at random. Find the probability of getting
ⓐ Exactly 2 tails ⓑ Atleast 2 tails ⓒ Atmost 2 tails.

Sol: when 3 coins are tossed at random, sample space is S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}

∴ Total possible cases $(n) = 2^3 = 8$

ⓐ Let 'A' be the event of getting exactly 2 tails.

∴ A = {TTH, THT, HTT}

∴ Favourable cases (m) = 3

∴ $P(A) = \frac{m}{n} = 3/8$

ⓑ Let 'B' be the event of getting atleast 2 tails

∴ B = {TTT, TTH, THT, HTT}

∴ Favourable cases (m) = 4.

∴ $P(B) = \frac{m}{n} = 4/8 = 1/2$

ⓒ Let 'C' be the event of getting atmost 2 tails

∴ C = {HHH, HHT, HTH, THH, TTH, THT, HTT}

∴ Favourable cases (m) = 7

∴ $P(C) = \frac{m}{n} = 7/8$

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② 2 letters are selected from the word 'GOAL'. Find the probability of getting
ⓐ Both vowels ⓑ one vowel and one consonant
ⓒ Both consonants ⓓ Atleast 1 consonants
ⓔ Atmost 1 consonants.

Sol: Since 2 letters are selected from the word 'GOAL'.

Sample space is S = {GO, GA, GL, OA, OL, AL}

∴ Total possible cases (n) = 6

ⓐ Let 'A' be the event of getting both vowels.

∴ A = {OA}.

∴ Favourable cases (m) = 1.

∴ $P(A) = m/n = 1/6$

(b) Let 'B' be the event of getting one vowel and one consonant.

$\therefore B = \{GO, GA, OL, AL\}$ $\therefore P(B) = \frac{m}{n} = 4/6 = 2/3$

$\therefore$ favourable cases(m) = 4

(c) Let 'C' be the event of getting both consonants.

$\therefore C = \{GL\}$ $\therefore P(C) = \frac{m}{n} = 1/6$

$\therefore$ favourable cases(m) = 1

(d) Let 'D' be the event of getting atleast 1 consonant

$\therefore D = \{GO, GA, GL, OL, AL\}$ $\therefore P(D) = \frac{m}{n} = 5/6$

$\therefore$ favourable cases (m) = 5

(e) Let 'E' be the event of getting atmost 1 consonant

$\therefore E = \{GO, GA, OA, OL, AL\}$ $\therefore P(E) = \frac{m}{n} = 5/6$

$\therefore$ favourable cases(m) = 5

(9) 1 student is selected at random out of 25 students in a class. Find the probability of getting a student bearing roll number -

(i) Multiple of 3 or 4 (ii) Multiple of 3 and 4

(iii) Not less than 21 (iv) Not more than 4

(v) which is prime (vi) which is composite.

(vii) which is neither prime nor composite.

soln Sample space of selecting a student out of 25 students is

$S = \{1, 2, 3, \ldots, 23, 24, 25\}$

(i) Let 'A' be the event of getting a student whose roll number is multiple of 3 or 4.

$\therefore A = \{3, 6, 9, 12, 15, 18, 21, 24, 4, 8, 16, 20\}$

$\therefore$ favourable cases(m) = 12

$\therefore P(A) = \frac{m}{n} = 12/25$

(ii) Let 'B' be the event of getting a student bearing roll number multiple of 3 and 4.

$\therefore B = \{12, 24\}$

$\therefore$ favourable cases(m) = 2

$\therefore P(B) = m/n = 2/25$

(iii) $C = \{21, 22, 23, 24, 25\}$

∴ ~~Fela~~ favourable no. of cases is $m = 5$

∴ $P(C) = \frac{m}{n} = \frac{5}{25} = \frac{1}{5}$.

(iv) $D = \{1, 2, 3, 4\}$

∴ favourable no. of cases is $m = 4$

∴ $P(D) = \frac{m}{n} = \frac{4}{25}$.

(v) $E = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

∴ favourable no. of cases is $m = 9$

∴ $P(E) = \frac{m}{n} = \frac{9}{25}$.

(vi) $F = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$

∴ favourable no. of cases is $m = 15$

∴ $P(F) = \frac{m}{n} = \frac{15}{25} = \frac{3}{5}$.

(vii) $G = \{1\}$

∴ favourable no. of cases is $m = 1$

∴ $P(G) = \frac{m}{n} = \frac{1}{25}$.

Q) A bag contains 4 red, 6 black and 7 green marbels. 3 marbels are selected at random. Find the probability of getting–

(a) All green marbels.

(b) 2 red and 1 black marbels.

(c) Marbels of same colours.

(d) " " different "

(e) Atleast 2 red marbels.

Soln Total Numbers of Marbels $= 4+6+7 = 17$

Since 3 marbels are selected at random.

∴ Total No. of Possible Cases is $n = {}^{17}C_3$

$= \frac{17 \times \cancel{16}^{8} \times \cancel{15}^{5}}{\cancel{3} \times \cancel{2} \times 1}$

$= 680$

(a) Let A be the event of getting all green marbels.
∴ Favourable no. of cases is $m = {}^7C_3$

$$= \frac{7\times6\times5}{3\times2\times1}$$

$$= 35.$$

$$\therefore P(A) = \frac{m}{n} = \frac{35}{680}$$

(b) Let B be the event of getting 2 red and 1 black marbels.
∴ Favourable no. of cases is $m = {}^4C_2 \times {}^6C_1$

$$= \frac{\overset{2}{\cancel{4}}\times3}{\cancel{2}\times1}\times6$$

$$= 36$$

$$\therefore P(B) = \frac{m}{n} = \frac{36}{680}.$$

(c) Let C be the event of getting marbels of same colour.
∴ Favourable no. of cases is $m = {}^4C_3 + {}^6C_3 + {}^7C_3$

$$= \frac{4\times\cancel{3}\times\cancel{2}}{\cancel{3}\times\cancel{2}\times1} + \frac{\cancel{6}\times5\times4}{\cancel{3}\times\cancel{2}\times1} + \frac{7\times\cancel{6}\times5}{\cancel{3}\times\cancel{2}\times1}$$

$$= 4 + 20 + 35 = 59$$

$$\therefore P(C) = \frac{m}{n} = \frac{59}{680}.$$

(d) Let D be the event of getting marbels of different colours.
∴ Favourable no. of cases is $m = {}^4C_1 \times {}^6C_1 \times {}^7C_1$

$$= 4\times6\times7$$

$$= 168$$

$$\therefore P(D) = \frac{m}{n} = \frac{168}{680}.$$

(c) Let E be the event of getting atleast 2 red marbles.

∴ favourable no. of cases is m =

Red(4)	Non Red (6+7=13)	No. of ways
2	1	$4c_2 \times 13c_1 = \frac{\overset{2}{\cancel{4}} \times 3}{2 \times 1} \times 13 = 78$
3	0	$4c_3 = 4 \times 1 = 4$
		82

$\therefore P(E) = \frac{m}{n} = \frac{82}{680}$.

(5) 2 dice are thrown at random. Find the probability of getting-

(a) A doublet.

(b) A total of 4 or 8 points.

(c) A total of atleast 10 "

(d) " " " atmost 5 "

(e) A difference of 4 points.

(f) A prime no. on 1st dice and a composite no. on 2nd dice.

(g) A prime no. on one " " " " " on the other

(h) Atleast one six.

Soln

Since 2 dice are thrown at random. Therefore Sample space is

$S = \{(1,1), \ldots\ldots (1,6)$

$(2,1), \ldots\ldots (2,6)$

$(3,1), \ldots\ldots (3,6)$

$(4,1), \ldots\ldots (4,6)$

$(5,1), \ldots\ldots (5,6)$

$(6,1), \ldots\ldots (6,6)\}$.

∴ Total no. of possible cases is $n = 36$.

(a) Let A be the event of getting a doublet.

$\therefore$ ~~Set~~ $A = \{(1,1), (2,2), (3,3), (4,4), \cancel{(5,5)}\ (5,5), (6,6)\}$

$\therefore$ Total no. of favourable cases is $m=6$.

$\therefore P(A) = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$

(b) Let B be the event of getting a total of 4 or 8 points.

$\therefore B = \{(1,3), (2,2), (2,6), (3,1), (3,5), (4,4), (5,3), (6,2)\}$

$\therefore$ Total no. of favourable cases is $m=8$

$\therefore P(B) = \frac{m}{n} = \frac{8}{36} = \frac{2}{9}$.

(c) Let C be the event of getting a total of atleast 10 points.

$\therefore C = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$\therefore$ Total no. of favourable cases is $m=6$

$\therefore P(C) = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$.

(d) Let D be the event of getting a ~~to~~total of atmost 5 points.

$\therefore D = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

$\therefore$ Favourable no. of cases is $m = \cancel{12}\ 10$

$\therefore P(D) = \frac{m}{n} = \frac{\cancel{10}^{5}}{\cancel{36}_{18}}$

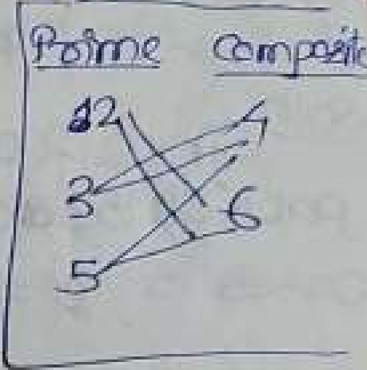
(e) Let E be the event of getting a difference of 4 points.

$\therefore E = \{(1,5), (2,6), (6,2), (5,1)\}$.

$\therefore$ Favourable no. of cases is $m=4$

$\therefore P(E) = m/n = 4/36 = 1/9$.

(f) Let F be the event of getting prime no. on 1st dice and composite no. on 2nd dice



$\therefore F = \{(2,4), (2,6), (3,4), (3,6), (5,4), (5,6)\}$.

$\therefore$ favourable no. of cases is $m = 6$.

$\therefore P(F) = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$.

(g) Let ~~B~~ G be the event of getting prime no. on one dice and composite no. on the other.

$\therefore G = \{(2,4)(2,6), (3,4), (3,6), (5,4), (5,6), (4,2), (6,2) (4,3), (6,3), (4,5), (6,5)\}$.

$\therefore$ favourable no. of cases is $m = 12$.

$\therefore P(G) = \frac{m}{n} = \frac{12}{36} = \frac{1}{3}$

(h) Let H be the event of getting atleast one six.

$\therefore H = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,1), (6,2), (6,3) (6,4), (6,5) (6,6)\}$

$\therefore$ favourable no. of cases is $m = 11$

$\therefore P(H) = \frac{m}{n} = \frac{11}{36}$.

Dt: 6/4/16

(6) Two cards are drawn at random from a pack of cards. Find the probability of getting:

(a) Both Kings.

(b) Both Spades.

(c) Both Non-Spade Cards.

(d) Both numbered Cards.

(e) Both face Cards.

(f) 1 King and 1 Ace.

(g) One numbered card and one face card.

(h) 1 spade and 1 club

(i) Cards of same colour.
(j) " " different "

Soln Since 2 cards are drawn at random from a pack of 52 cards. So, total number of possible cases is $n = {}^{52}C_2$

$$= \frac{52 \times 51}{2} = 1326$$

(a) Let 'A' be the event of getting both kings.

∴ favourable no. of cases is $m = {}^{4}C_2$

$$= \frac{\overset{2}{\cancel{4}} \times 3}{2} = 6$$

$$\therefore P(A) = \frac{m}{n} = \frac{6}{1326}$$

(b) Let 'B' be the event of getting both spades.

∴ favourable no. of cases is $m = {}^{13}C_2$

$$= \frac{13 \times \overset{6}{\cancel{12}}}{2 \times 1} = 78$$

$$\therefore P(B) = \frac{m}{n} = \frac{78}{1326}$$

(c) Let 'C' be the event of getting both non-spade cards.

∴ favourable no. of cases is $m = {}^{39}C_2$

$$= \frac{39 \times \overset{19}{\cancel{38}}}{2} = 741$$

$$\therefore P(C) = \frac{m}{n} = \frac{741}{1326}$$

(d) Let 'D' be the event of getting both numbered cards.

∴ favourable no. of cases is $m = {}^{36}C_2 = \frac{\overset{18}{\cancel{36}} \times 35}{2} = 630$

$$\therefore P(D) = \frac{m}{n} = \frac{630}{1326}$$

(e) Let 'E' be the event of getting both face cards.
∴ Favourable no. of cases is $m = {}^{12}C_2 = \frac{12\times 11}{2} = 66$

$\therefore P(E) = \frac{m}{n} = \frac{66}{1326}$

(f) Let 'F' be the event of getting 1 King and 1 Ace.
∴ Favourable no. of cases is $m = {}^{4}C_1 \times {}^{4}C_1 = 4\times 4 = 16$

$\therefore P(F) = \frac{m}{n} = \frac{16}{1326}$

(g) Let 'G' be the event of getting 1 numbered card and 1 face card.
∴ Favourable no. of cases is $m = {}^{36}C_1 \times {}^{12}C_1 = 36\times 12 = 432$

$\therefore P(G) = \frac{m}{n} = \frac{432}{1326}$

(h) Let 'H' be the event of getting 1 spade and 1 club.
∴ Favourable No. of cases is $m = {}^{13}C_1 \times {}^{13}C_1 = 13\times 13 = 169$

$\therefore P(H) = \frac{m}{n} = \frac{169}{1326}$

(i) Let 'I' be the event of getting cards of same colour i.e, either both red or both black.
∴ Favourable no. of cases is $m = {}^{26}C_2 + {}^{26}C_2$

$= 2\times\frac{26\times 25}{2\times 1} = 650$

$\therefore P(I) = \frac{m}{n} = \frac{650}{1326}$

(j) Let 'J' be the event of getting cards of different colours i.e, one red and one black.
∴ Favourable no. of cases is $m = {}^{26}C_1 \times {}^{26}C_1$

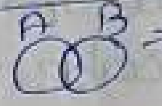
$= 26\times 26 = 676$

$\therefore P(J) = \frac{m}{n} = \frac{676}{1326}$

(7) State generalised additional theorem of probability. What happens if the events are mutually exclusive?

→ If A and B be any two non-mutually exclusive events of a random experiment then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



→ common Element

If A and B be any two mutually exclusive events of a random experiment then

$P(A \cup B) = P(A) + P(B)$


→ No common element

---

(8) State multiplicative law of probability.

→ If A and B be any two dependent events then

$P(A \cap B) = P(A) \times P(B/A)$ → Given

'OR'

$= P(B) \times P(A/B)$

If A and B be any two independent events then $P(A \cap B) = P(A) \times P(B)$

---

(9) If $P(A \cup B) = 0.42$, $P(A) = 0.31$, $P(B) = 0.24$. Find-

(a) $P(A \cap B)$

(b) $P(\text{only } A)$ or $P(A \text{ but not } B)$ or $P(A \cap \bar{B})$ or $P(A \cap B')$

(c) $P(\overline{A \cup B})$ or neither A nor B.

(d) $P(A/B)$ (e) $P(B/A)$ (f) $P(A/B^c)$ (g) $P(A^c/B^c)$

(h) $P(A^c/B)$

Soln

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.31 + 0.24 - 0.42 = 0.13$

(b) $P(\text{only } A) = P(A) - P(A \cap B)$

$= 0.31 - 0.13 = 0.18$

(c) $P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - 0.42 = 0.58$

(d) $P(A\cap B) = P(B) \times P(A/B)$

$\Rightarrow 0.13 = 0.24 \times P(A/B)$

$\Rightarrow 0.54 = P(A/B)$

(e) $P(A\cap B) = P(A) \times P(B/A)$

$\Rightarrow 0.13 = 0.31 \times P(B/A)$

$\Rightarrow 0.42 = P(B/A)$

(f) $P(A/B^c) \times P(B^c) = P(A\cap B^c)$

$\Rightarrow 0.18 = (1-0.24) \times P(A/B^c)$

$\Rightarrow \frac{0.18}{0.76} = P(A/B^c)$

$\Rightarrow 0.236 = P(A/B^c)$

De Morgan's Law:

$(A\cup B)^c = A^c \cap B^c$

$(A\cap B)^c = A^c \cup B^c$

(g) $P(A^c/B^c) \times P(B^c) = P(A^c \cap B^c)$

$\Rightarrow P(A\cup B)^c = 0.76 \times P(A^c/B^c)$

$\Rightarrow \frac{1-0.42}{0.76} = P(A^c/B^c)$

$\Rightarrow \frac{0.58}{0.76} = P(A^c/B^c)$

$\Rightarrow 0.76 = P(A^c/B^c)$

(h) $P(A^c/B) \times P(B) = P(A^c \cap B)$

$\Rightarrow \frac{0.24-0.13}{0.24} = P(A^c \cap B)$

$\Rightarrow \frac{0.11}{0.24} = P(A^c \cap B)$

$\Rightarrow 0.458 = P(A^c \cap B)$.

(10) The probability that Amal and Bimal can get a job is 32% and 45% respectively. Find the probability that-

(a) Both of them will get the job)

(b) None will get the job

ⓒ Atleast one of them will get the job

'OR'

Either Amal or Bimal " " " ".

Soln

Let A and B be the event of getting a job by Amal and Bimal respectively.

B/Q $P(A) = 32\% = 0.32$; $P(\bar{A}) = 1-0.32 = 0.68$

$P(B) = 45\% = 0.45$; $P(\bar{B}) = 1-0.45 = 0.55$

ⓐ P(Both will get the job)

$= P(A \cap B)$

$= P(A) \times P(B)$ [∵ A and B are independent events]

$= 0.32 \times 0.45 = 0.144$.

ⓑ P(None will get the job)

$= P(A \cup B)'$

$= P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \times P(\bar{B})$

$= \{1-P(A)\} \times \{1-P(B)\}$

$= P(\bar{A}) \times P(\bar{B})$

$= 0.68 \times 0.55$

$= 0.374$

$= (1-0.32) \times (1-0.45) = 0.68 \times 0.55 = 0.374$

ⓒ P(Either Amal or Bimal will get the job)

$= P(A \cup B)$

$= 1 - P(A \cup B)'$

$= 1 - 0.374 = 0.626$

$= 1 - [P(\bar{A}) \times P(\bar{B})]$

$= 1 - [0.68 \times 0.55]$

$= 1 - 0.374 = 0.626$

---

⑪ Amal, Bimal, Kamal and Shamal can solve 20%, 25%, 15%, 8% respectively a problem of statistics. Find the probability that–

ⓐ All of them can solve the problem.

ⓑ None " " " " ".

ⓒ Atleast one of them can solve the problem.

Soln

Let A, B, C and D be the event of solving a problem of statistics by Amal, Bimal, Kamal and Shamal respectively.

$\therefore P(A) = 20\% = 0.20$; $P(\bar{A}) = 1-0.20 = 0.80$

$P(B) = 25\% = 0.25$; $P(\bar{B}) = 1-0.25 = 0.75$

$P(C) = 15\% = 0.15$; $P(\bar{C}) = 1-0.15 = 0.85$

$P(D) = 8\% = 0.08$; $P(\bar{D}) = 1-0.08 = 0.92$

(a) P(All of them can solve the problem)

$= P(A \cap B \cap C \cap D)$ [A, B, C, D are independent events]

$= P(A) \times P(B) \times P(C) \times P(D)$

$= 0.20 \times 0.25 \times 0.15 \times 0.08$

$= 0.0006$

(b) P(None of them can solve the problem)

$= P(A \cup B \cup C \cup D)'$

$= P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) \times P(\bar{D})$

$= \{1-P(A)\} \times \{1-P(B)\} \times \{1-P(C)\} \times \{1-P(D)\}$

$= \{1-0.20\} \times \{1-0.25\} \times \{1-0.15\} \times \{1-0.08\}$

$= 0.80 \times 0.75 \times 0.85 \times 0.92 = 0.4692$

$1 - P(A \cup B \cup C \cup D)$
$= 1 - 0.5308$
$= 0.4692$

(c) P(Atleast one of them can solve the problem)

$= P(A \cup B \cup C \cup D)$

$= 1 - P(A \cup B \cup C \cup D)'$

$= 1 - 0.4692 = 0.5308$

$1 - [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) \times P(\bar{D})]$
$= 1 - [0.80 \times 0.75 \times 0.85 \times 0.92]$
$= 1 - 0.4692$
$= 0.5308$

D-18/4/16

(12) Given, $P(A \cup B) = 0.84$, $P(A) = 0.25$ and A, B are independent events. Find $P(B)$.

Soln

We know, $P(A \cap B) = P(A) \times P(B)$ [$\because$ A, B are independent events]

$\Rightarrow P(A) + P(B) - P(A \cup B) = P(A) \times P(B)$

$\Rightarrow 0.25 + P(B) - 0.84 = 0.25 \times P(B)$

$\Rightarrow P(B) - 0.25P(B) = 0.59$

$\Rightarrow P(B) = \frac{0.59}{0.75} = 0.78$

(13) If A,B,C are mutually exclusive and exhaustive events of a random experiment and $P(A) = \frac{1}{2}P(B) = \frac{2}{3}P(C)$. Find $P(A), P(B), P(C)$

[A B C No common element]

[$A \cup B \cup C = U$]

Soln

Since A,B,C are mutually exclusive and exhaustive events. So,

$P(A \cap B \cap C) = 0$; $P(A \cap B) = 0$; $P(A \cap C) = 0$, $P(B \cap C) = 0$ and $P(A \cup B \cup C) = 1$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) = 1$

$\Rightarrow P(A) + P(B) + P(C) - 0 - 0 - 0 = 1$

$\Rightarrow P(A) + 2P(A) + \frac{3}{2}P(A) = 1$

$\Rightarrow P(A)\left(1 + 2 + \frac{3}{2}\right) = 1$

$\Rightarrow P(A)(1 + 2 + 1.5) = 1$

$\Rightarrow P(A) = \frac{1}{4.5} = \frac{2}{9}$

$\therefore P(B) = 2P(A) = 2 \times \frac{2}{9} = \frac{4}{9}$

$P(C) = \frac{3}{2} \times P(A) = \frac{3}{2} \times \frac{2}{9} = \frac{3}{9}$

Binomial Distribution:

Binomial Probability Mass Function (P.m.f) is

$P(x) = {}^nC_x \times p^x \times q^{n-x}$

where, n = No. of trials

x = No. of success (0,1,2, -- n)

p = Probability of success

q = " " failures.

$p + q = 1$

i) Mean = np ii) S.D = $\sqrt{npq}$

iii) Variance = npq.

(14) If 8 coins are tossed at random. Find the probability of getting

a) Exactly 3 heads or 5 tails
b) Atleast 7 "
c) Atmost 2 "
d) Atleast 1 "
e) Atmost 7 "

Also find mean, S.D, Variance.

Soln

Here $n=8$

p = Probability of getting a head $= \frac{1}{2} = 0.5$

q = " " " tail $= 1-0.5 = 0.5$

We know Binomial p.m.f is

$p(x) = {}^nC_x \times p^x \times q^{n-x}$

$= 8C_x \times (0.5)^x \times (0.5)^{8-x}$

i) P(exactly 3 heads)

$= P(x=3)$

$= 8C_3 \times (0.5)^3 \times (0.5)^{8-3}$

$= \frac{8\times7\times6}{3\times2\times1} \times (0.5)^{3+5}$

$= 56 \times (0.5)^8 = 56 \times 0.0039062 = 0.2187472$

ii) P(Atleast 7 heads)

$= P(x=7) + P(x=8)$

$= 8C_7 \times (0.5)^7 \times (0.5)^{8-7} + 8C_8 \times (0.5)^8 \times (0.5)^{8-8}$

$= 8 \times (0.5)^8 + 1 \times (0.5)^8 \times 1$

$= 0.0039062 \times 9 = 0.0351558$

iii) P(Atmost 2 heads)

$= P(x=0) + P(x=1) + P(x=2)$

$= {}^8C_0 \times (0.5)^0 \times (0.5)^{8-0} + {}^8C_1 \times (0.5)^1 \times (0.5)^{8-1} + {}^8C_2 \times (0.5)^2 \times (0.5)^6$

$= 1 \times (0.5)^8 + 8 \times (0.5)^8 + \frac{8 \times 7}{2 \times 1} \times (0.5)^8$

$= (0.5)^8 [1 + 8 + 28]$

$= 37 \times 0.0039062 = 0.1445294$

ii) $P(\text{atleast 1 head})$

$= 1 - P(x=0)$

$= 1 - {}^8C_0 \times (0.5)^0 \times (0.5)^{8-0}$

$= 1 - 1 \times 0.0039062 = 0.9960938$

v) $P(\text{atmost 7 heads})$

$= 1 - P(x=8)$

$= 1 - {}^8C_8 \times (0.5)^8 \times (0.5)^{8-8}$

$= 1 - 1 \times 0.0039062 = 0.9960938$

Now, Mean $= np = 8 \times 0.5 = 4$

$S.D = \sqrt{npq}$

$= \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{2} = 1.414$

Variance $= (\sqrt{2})^2 = 2$.

(15) If Mean and S.D of a binomial distribution are 7.5 and 1.37 respectively. Find n, p, q and write B.D (Binomial Distribution)

Soln

Given,

Mean $= 7.5$

$\Rightarrow np = 7.5 \rightarrow (1)$

$S.D = 1.37$

$\Rightarrow \sqrt{npq} = 1.37$

$\Rightarrow npq = 1.8769$

$\Rightarrow 7.5q = 1.8769$ [From (1)]

$\Rightarrow q = 0.2503$

Again, $p+q=1$

$\Rightarrow p+0.2503=1$

$\Rightarrow p=1-0.2503=0.7497$

From ①

$n\times 0.7497=7.5$

$\therefore n=10$

$\therefore$ Required Binomial Distribution is

$^{10}C_x \times (0.7497)^x \times (0.2503)^{10-x}$

where $x=0,1,2,\ldots,10$

---

(16) Out of 6 workers in a firm, there is a chance of 20% having occupational disease. Find the probability of getting —

i) exactly 4 sufferers

ii) Atleast 5 "

iii) Atmost 2 "

iv) Atleast 1 "

v) Atmost 4 "

Also find Mean, S.D and variance.

Soln

Here, $n=6$

p = Probability of getting a worker having occupational disease $=\frac{20}{100}=0.20$

q = Probability of failure $=1-0.20=0.80$

We know

Binomial p.m.f is

$^6C_x \times (0.20)^x \times (0.80)^{6-x}$

i) P(exactly 4 sufferers)

$=P(x=4)$

$= {}^6C_4 \times (0.20)^4 \times (0.80)^{6-4}$

$=\frac{\overset{3}{\cancel{6}}\times 5}{\cancel{2}\times 1}\times 0.0016 \times 0.64 = 0.01536$

ii) P(atleast 5 successes)

$= P(x=5) + P(x=6)$

$= {}^6C_5 \times (0.20)^5 \times (0.80)^{6-5} + {}^6C_6 \times (0.20)^6 \times (0.80)^{6-6}$

$= 0.001536 + 0.000064 = 0.0016.$

iii) P(atmost 2 successes)

$= P(x=0) + P(x=1) + P(x=2)$

$= {}^6C_0 \times (0.20)^0 \times (0.80)^{6-0} + {}^6C_1 \times (0.20)^1 \times (0.80)^{6-1} + {}^6C_2 \times (0.20)^2 \times (0.80)^{6-2}$

$= 1 \times 1 \times 0.262144 + 6 \times 0.20 \times 0.32768 + \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2} \times 1} \times \overset{0.04}{\cancel{04}} \times 0.4096$

$= 0.262144 + 0.393216 + 0.24576$

$= 0.89112$

iv) P(atleast 1 successes)

$= 1 - P(x=0)$

$= 1 - {}^6C_0 \times (0.20)^0 \times (0.80)^{6-0}$

$= 1 - .262144 = 0.737856$

v) P(atmost 4 successes)

$= 1 - P(x=5) - P(x=6)$

$= 1 - {}^6C_5 \times (0.20)^5 \times (0.80)^{6-5} - {}^6C_6 \times (0.20)^6 \times (0.80)^{6-6}$

$= 1 - 6 \times 0.80 \times 0.00032 - 1 \times 1 \times 0.000064$

$= 1 - 0.000064 - 0.001536$

$= 0.9984$

Dt 21/4/16

Poisson P.m.f is

$$P(x) = \frac{e^{-m} \times m^x}{\lfloor x} \qquad [e = 2.71]$$

where $m = \text{mean} = np$

Note: Mean = Variance = m
$SD = \sqrt{m}$.

(10) If 2% of electric bulbs manufactured by a Co. is defective find the probability that out of 400 electric bulbs -

i) No bulb is defective.
ii) exactly 2 bulbs are defective.
iii) Atmost 3 " " "
iv) Atleast 2 " " " $[e^{-8} = 0.0003]$

Soln Here $n = 400$

p = Probability of getting a defective bulb $= 2\% = \frac{2}{100}$

$\therefore m = np = 400 \times \frac{2}{100} = 8$

we know Poisson p.m.f is

$$P(x) = \frac{e^{-m} \times m^x}{\lfloor x}$$

$$= \frac{e^{-8} \times 8^x}{\lfloor x}$$

i) P(no defective bulb)

$= P(x=0)$

$$= \frac{e^{-8} \times 8^0}{\lfloor 0} = \frac{e^{-8} \times 1}{1} = e^{-8} = (2.71)^{-8} = \frac{1}{(2.71)^8} = 0.0003$$

ii) P(exactly 2 defective bulbs)

$= P(x=2)$

$$= \frac{e^{-8} \times 8^2}{\lfloor 2} = 0.0003 \times \frac{8 \times 8^4}{2 \times 1} = 0.0096$$

iii) P(atmost 3 defective bulbs)

$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= \frac{e^{-8} \times 8^0}{\lfloor 0} + \frac{e^{-8} \times 8^1}{\lfloor 1} + \frac{e^{-8} \times 8^2}{\lfloor 2} + \frac{e^{-8} \times 8^3}{\lfloor 3}$$

$$= e^{-8}\left[1 + 8 + \frac{8 \times 8^4}{2 \times 1} + \frac{8 \times 8^4 \times 8}{3 \times 2 \times 1}\right]$$

$= 0.0003[9 + 32 + 85.33]$

$= 0.003 \times 126.33 = 0.378$

iv) P(atleast 2 defective bulbs)

$= 1 - P(x=0) - P(x=1)$

$$= 1 - \frac{e^{-8} \times 8^0}{\lfloor 0} - \frac{e^{-8} \times 8^1}{\lfloor 1}$$

$= 1 - e^{-8}(1 + 8)$

$= 1 - 0.003 \times 9 = 1 - 0.027 = 0.973$

---

(18) The poisson distribution has double modes at $x=1$ and $x=3$ find

i) Mean ii) Variance iii) S.D.

Also write poisson Distribution.

Soln Since the distribution has double modes at $x=1$ and $x=3$

$\therefore P(x=1) = P(x=3)$

$$\Rightarrow \frac{e^{-m} \times m^x}{\lfloor x} = \frac{e^{-m} \times m^x}{\lfloor x}$$

$$\Rightarrow \frac{e^{-m} \times m^1}{\lfloor 1} = \frac{e^{-m} \times m^3}{\lfloor 3}$$

$$\Rightarrow \frac{m}{1} = \frac{m^3}{6} m^2$$

$\Rightarrow m = 2.45$

i) Mean $= m = 2.45$

ii) Variance $= m = 2.45$

iii) S.D $= \sqrt{2.45} = 15652$

$\therefore$ poisson distribution is

$$\frac{e^{-2.45} \times (2.45)^x}{\lfloor x}$$

(19) Bring out the fallacy (if any)

(a) In Poisson distribution, Mean = 5, S.D = $\sqrt{5}$.

(b) " " , Mean = 5, S.D = 2.33

Soln

(a) Given, Mean = 5

S.D = $\sqrt{5}$.

$\therefore$ Variance = $(S.D)^2 = (\sqrt{5})^2 = 5$

Since Mean = Variance

$\therefore$ Given statement is correct.

(b) Given, Mean = 5

S.D = 2.33

$\therefore$ Variance = $(S.D)^2 = (2.33)^2 = 5.4289$

Since Mean $\neq$ Variance

$\therefore$ Given statement is incorrect.

(20) What is the probability of getting in a leap year -

i) 53 mondays.

ii) 53 " or tuesdays.

iii) 53 " " wednesdays.

iv) 53 " and tuesdays.

Soln We know that a leap year consists of 366 days i.e, 52 weeks with 2 extra days. These extra 2 days can be arranged as follows:

1) (Sunday, Monday).

2) (Monday, Tuesday).

3) (Tuesday, Wednesday).

4) (Wednesday, Thursday).

5) (Thursday, Friday).

6) (Friday, Saturday).

7) (Saturday, Sunday).

$\therefore$ Total Possible cases is (n) = 7.

i) Let 'A' be the event of getting 53 Mondays.

∴ Favourable No. of cases $(m) = 2$

$\therefore P(A) = \frac{m}{n} = \frac{2}{7}$.

ii) Let 'B' be the event of getting 53 Mondays or 53 Tuesdays.

∴ Favourable No. of cases $(m) = 3$

$\therefore P(B) = \frac{m}{n} = \frac{3}{7}$.

iii) Let 'C' be the event of getting 53 Mondays or 53 wednesdays.

∴ Favourable No. of cases $(m) = 4$

$\therefore P(C) = \frac{m}{n} = \frac{4}{7}$.

iv) Let 'D' be the event of getting 53 Mondays and 53 Tuesdays.

∴ Favourable No. of cases $(m) = 1$

$\therefore P(D) = \frac{m}{n} = \frac{1}{7}$.

Baye's Theorem

⊛ Write the statement of Baye's Theorem.

→ If $E_1, E_2, \ldots E_n$ be n- mutually exclusive and exhaustive events of a random experiment and D be the arbitrary event then

$$P(E_i/D) = \frac{P(E_i) \times P(D/E_i)}{P(E_1) \times P(D/E_1) + P(E_2) \times P(D/E_2) + \ldots + P(E_n) \times P(D/E_n)}$$

where, $i = 1, 2, 3, 4, \ldots n$

(2) A company has 3 machines X, Y, Z which produce 4000, 5000 and 9000 units of T.V's. If machine X, Y, Z produce 2%, 3% and 6% defective T.V's. Find the probability of getting a defective T.V

(a) Produced by machine X from the total output?

(b) " " " Y " " " " ?

(c) " " " Z " " " " ?

(d) " " either X or Y?

Sol^n Let E_1, E_2, E_3 be the events of producing T.V's and D be the event of getting a defective T.V.

$P(E_1) = \frac{4000}{18000} = 0.22$

$P(D/E_1) = 2\% = 0.02$

$P(D/E_2) = 3\% = 0.03$

$P(E_2) = \frac{5000}{18000}$

$P(D/E_3) = 6\% = 0.06$

$= 0.277$

$P(E_3) = \frac{9000}{18000} = 0.50$

i) $P(E_1/D) = \frac{P(E_1) \times P(D/E_1)}{P(E_1) \times P(D/E_1) + P(E_2) \times P(D/E_2) + P(E_3) \times P(D/E_3)}$

$= \frac{0.22 \times 0.02}{0.22 \times 0.02 + 0.277 \times 0.03 + 0.50 \times 0.06}$

$= \frac{0.0044}{0.04271}$

$= 0.10$

ii) $P(E_2/D) = \frac{0.277 \times 0.03}{0.04271}$

$= \frac{0.00831}{0.04271}$

$= 0.19$

iii) $P(E_3/D) = \frac{0.50 \times 0.06}{0.04271}$

$= \frac{0.03}{0.04271} = 0.70$

iv) Required probability is

$P(E_1/D) + P(E_2/D)$

$= 0.10 + 0.19$

$= 0.29$

Dt 13/4/16

(22) Given, Urn1 : 4 red, 5 white balls.

Urn2 : 6 red, 7 " "

Urn3 : 5 red, 8 " "

If one of the urn is selected at random and 1 white ball is drawn from it, find the probability that it has come from -

(a) 1st Urn.

(b) 2nd Urn.

(c) 3rd Urn.

Soln Let E_1, E_2, E_3 be the events of selecting 1st Urn, 2nd Urn, 3rd Urn respectively and W be the event of getting a white ball.

B/P $P(E_1) = \frac{1}{3}$ $\quad P(W/E_1) = \frac{5}{4+5} = \frac{5}{9}$

$P(E_2) = \frac{1}{3}$ $\quad P(W/E_2) = \frac{7}{6+7} = \frac{7}{13}$

$P(E_3) = \frac{1}{3}$ $\quad P(W/E_3) = \frac{8}{5+8} = \frac{8}{13}$

(a) $P(E_1/W) = \frac{P(E_1) \times P(W/E_1)}{P(E_1) \times P(W/E_1) + P(E_2) \times P(W/E_2) + P(E_3) \times P(W/E_3)}$

$= \frac{0.33 \times 0.55}{0.33 \times 0.55 + 0.33 \times 0.538 + 0.33 \times 0.615}$

$= \frac{0.1833}{0.5621} = 0.326$

(b) $P(E_2/W) = \frac{0.33 \times 0.538}{0.5621} = 0.316$

(c) $P(E_3/W) = \frac{0.33 \times 0.615}{0.5621}$

$= 0.361$

Index Number

Dt: 25/3/16

① Determine Price Indices to the following years with respect to

ⓐ 1995 as base year.

ⓑ 1997 as base year

Year :	1993	94	95	96	97	98	99	2000
Prices :	40	50	55	42	47	80	85	90

Soln

Calculation of Price Indices

Years	Prices	Price Index: Base Year 1995	Price Index: Base Year 1997
1993	40	$\frac{40}{55} \times 100 = 72.72$	$40/47 \times 100 = 85.10$
1994	50	$50/55 \times 100 = 90.91$	$50/47 \times 100 = 106.38$
1995	55	$55/55 \times 100 = 100$	$55/47 \times 100 = 117.02$
1996	42	76.36	89.36
1997	47	85.45	100
1998	80	145.45	170.21
1999	85	154.54	180.85
2000	90	163.63	191.48

Rice

2015	2016
₹20	₹25 – Price
Base Year	Current Year
Notation '0'	Notation '1' or 'n'.

$P_0 = ₹20; \quad P_1 = ₹25$

$\therefore P_{01} = \frac{P_1}{P_0} \times 100 = \frac{25}{20} \times 100 = 125$

2) Determine Laspeyre's, Paasche's, Marshall-Edgeworth's, Fisher's price and Quantity indices from the data given:

(a) Commodities	Base Year Price	Base Year Qty	Current Year Price	Current Year Qty
A	10	4	15	3
B	12	3	30	2
C	20	5	25	4

Soln Let P_0 denotes price in base year.
P_1 " " " current "
Q_0 " quantity " base "
Q_1 " " " current "

Working Table

Commodities	P_0	Q_0	P_1	Q_1	P_0Q_0	P_0Q_1	P_1Q_0	P_1Q_1
A	10	4	15	3	40	30	60	45
B	12	3	30	2	36	24	90	60
C	20	5	25	4	100	80	125	100
					$\Sigma P_0Q_0 = 176$	$\Sigma P_0Q_1 = 134$	$\Sigma P_1Q_0 = 275$	$\Sigma P_1Q_1 = 205$

Laspeyre's Formula:

$$\text{Price Index } (P_{0,1}) = \frac{\Sigma P_1Q_0}{\Sigma P_0Q_0} \times 100$$

$$= \frac{275}{176} \times 100 = 156.25$$

Comment: $P_{0,1} = 156.25$ indicates that there is an increase of 56.25% in the prices of the commodities in current year w.r.t base Year.

Quantity Index $(Q_{01}) = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$

$= \frac{134}{176} \times 100 = 76.136$

Comment: $Q_{01} = 76.136$ indicates that there is a decrease of 23.864% in the quantity of the commodities in current year w.r.t. base year.

Paasche's Formula:

Price Index $(P_{01}) = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$

$= \frac{205}{134} \times 100 = 152.98$

Quantity Index $(Q_{01}) = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$

$= \frac{205}{275} \times 100 = 74.55$

Marshall-Edgeworth's Formula:

Price Index $(P_{01}) = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$

$= \frac{275+205}{176+134} \times 100$

$= \frac{480 \times 100}{310} = 154.84$

Quantity Index $(Q_{01}) = \frac{\sum q_1 p_0 + \sum q_1 p_1}{\sum q_0 p_0 + \sum q_0 p_1} \times 1$

$= \frac{134+205}{176+275} \times 100$

$= \frac{339 \times 100}{451} = 75.17$

Fisher's Formula:

$$\text{Price Index } (P_{0,1}) = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{275}{176} \times \frac{205}{134}} \times 100$$

$$= 1.54608 \times 100 = 154.608$$

$$\text{Quantity Index } (Q_{0,1}) = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

$$= \sqrt{\frac{134}{176} \times \frac{205}{275}} \times 100$$

$$= 0.7533 \times 100 = 75.3367$$

Dt: 30/3/16

③ Examine whether Laspeyre's and Fisher's formula satisfy
(i) Time Reversal Test (TRT)
(ii) Factor " " (FRT)

Commodities	2010 Price	2010 Qty	2011 Price	2011 Qty
A	20	5	30	4
B	25	4	40	3
C	30	2	50	2

Soln

Working Table

Commodities	p_0	q_0	p_1	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
A	20	5	30	4	100	80	150	120
B	25	4	40	3	100	75	~~16~~ 160	120
C	30	2	50	2	60	60	100	100
					$\sum p_0q_0 =$ 260	$\sum p_0q_1 =$ 215	$\sum p_1q_0 =$ 410	$\sum p_1q_1 =$ 340

i) Time Reversal Test is $P_{01} \times P_{10} = 1$.

Laspeyre's Formula:

L.H.S

$P_{0,1} \times P_{1,0}$ [In case of $P_{1,0}$, 0 will be replaced by 1, 1 " " " 0]

$= \frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_0 Q_1}{\Sigma P_1 Q_1}$ [omitting the factor 100]

$= \frac{410}{260} \times \frac{215}{340} = 0.9972$

∴ Laspeyre's Formula does not satisfy TRT.

Fisher's Formula:

L.H.S

$\sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times \sqrt{\frac{\Sigma P_0 Q_1}{\Sigma P_1 Q_1} \times \frac{\Sigma P_0 Q_0}{\Sigma P_1 Q_0}}$ [omitting factor 100]

$= \sqrt{\frac{410}{260} \times \frac{340}{215}} \times \sqrt{\frac{215}{340} \times \frac{260}{410}}$

$= \sqrt{\frac{410 \times 340 \times 215 \times 260}{260 \times 215 \times 340 \times 410}} = 1 = R.H.S$

∴ Fisher's Formula satisfy TRT.

ii) Factor Reversal Test is $P_{01} \times Q_{01} = \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$

Laspeyre's Formula:

L.H.S

$P_{01} \times Q_{01}$.

$= \frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0}$

$= \frac{410 \times 215}{260 \times 260} = 1.3039$

R.H.S

$\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$

$= \frac{340}{260}$

$= 1.3077$

∴ LHS ≠ R.H.S

∴ Laspeyre's formula does not satisfy FRT.

Fisher's formula :

L.H.S

$P_{0,1} \times Q_{0,1}$

$$= \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times \sqrt{\frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times \frac{\Sigma Q_1 P_1}{\Sigma Q_0 P_1}}$$

$$= \sqrt{\frac{410}{260} \times \frac{340}{215}} \times \sqrt{\frac{215}{260} \times \frac{340}{410}}$$

$$= \sqrt{\frac{410 \times 340 \times 215 \times 340}{260 \times 215 \times 260 \times 410}}$$

$$= \sqrt{\left(\frac{340}{260}\right)^2} = \frac{34}{26} = 1.3077 = R.H.S.$$

∴ Fisher's formula satisfy FRT.

④ Determine cost of Living Index (CLI) or consumer Price Index from the following data and comment on the result.

ⓐ Commodities	Price 2010	Price 2015	% Expenditure or weight
A	40	50	30
B	50	60	25
C	62	65	10
D	80	90	15
E	90	95	20

Mr Ram earns ₹20,000 in base year. How much should he earn in the current year so as to maintain the same standard of living as in the base year?

Soln

We know

$$CLI = \frac{\Sigma PW}{\Sigma W}$$

where, P = Price Index $= \frac{P_1}{P_0} \times 100$

W = % Expenditure or weight.

Working Table

Commodities	P_0	P_1	W	$P = \frac{P_1}{P_0} \times 100$	PW
A	40	50	30	$50/40 \times 100 = 125$	3750
B	50	60	25	$60/50 \times 100 = 120$	3000
C	62	65	10	104.83	1048.30
D	80	90	15	112.50	1687.50
E	90	95	20	105.56	211.20
			$\Sigma W = 100$		$\Sigma PW = 1159$

Now, $CLI = \frac{11597}{100} = 115.97$

Comment: CLI = 115.97 indicates that there is an increase of 15.97% in the prices of the commodities in current year w.r.t base year

Earning of Mr Ram in Base year = ₹20,000.

" " " " "current" $= ₹(20000 \times \frac{115.97}{100}$

$= ₹23194$

⑤ Determine real income or real wage from the following data:

ⓐ Year	Actual Income Money Income(₹)	Price Index
2000	4000	120
2001	5000	80
2002	6000	150
2003	5400	90
2004	6200	180
2005	7000	200

Also determine real income indices and

comment on the trend.

Soln We Know,

$$\text{Real Income} = \frac{\text{Money Income}}{\text{Price Index}} \times 100$$

$$\text{Real Income Index} = \frac{\text{Current Years Real Income}}{\text{Base Years Real Income}} \times 100$$

Table for calculation of Real Income and Real Income Indices

Years	Money Income (₹)	Price Index	Real Income	Real Income Index (Base Yr = 2000)
2000	4000	120	$\frac{4000}{120} \times 100 = 3333.33$	$\frac{3333.33}{3333.33} \times 100 = 100$
2001	5000	80	$\frac{5000}{80} \times 100 = 6250$	$\frac{6250}{3333.33} \times 100 = 187.50$
2002	6000	150	$\frac{6000}{150} \times 100 = 4000$	$\frac{4000}{3333.33} \times 100 = 120.00$
2003	5400	90	$\frac{5400}{90} \times 100 = 6000$	$\frac{6000}{3333.33} \times 100 = 180$
2004	6200	180	$6200/180 \times 100 = 3444.44$	103.33
2005	7000	200	$\frac{7000}{200} \times 100 = 3500$	105.

Comment : The trend is mixed.

Averages, Dispersion, Skewness and Kurtosis

- AM
- GM
- HM
- Median
- Mode

- Range
- Q.D
- M.D
- S.D

Absolute — concrete No (means with units).

C.Vof — Relative

$\Sigma \rightarrow$ Means summession
Known as capital sigma

$\sigma \rightarrow$ Mean S.D
Known as small sigma

change of origin means minus $(-)$
" " scale " divide $(\div)$

$$d = \frac{x - a}{h}$$

$a \rightarrow$ Deviation $\rightarrow$ Intermediate value.
$h \rightarrow$ common difference (step)

① Determine Mean (AM), Median, Mode, Range, S.D or (σ), Variance, Co-efficient of Range, Co-efficient of S.D, C.V, Co-efficient of skewness from the following data:

Q) Marks (x): 20, 15, 18, 8, 30, 12, 20, 26, 22, 13.

(Individual series or simple series).

Soln

Working Table

x	$d = \frac{x-a}{h}$	d^2
8	-10	100
12	-6	36
13	-5	25
15	-3	9
18	0	0
20	2	4
20	2	4
22	4	16

26	8	64
30	12	144
N=10	$\Sigma d=4$	$\Sigma d^2=402$

Here, a = 18, h = 1

Now, $A.M(\bar{x}) = a + \frac{\Sigma d}{n} \times h$

$= 18 + \frac{4}{10} \times 1 = 18.40$ Marks

Median = Value of $\left(\frac{N+1}{2}\right)^{th}$ observation

= " " $\left(\frac{10+1}{2}\right)^{th}$ "

= " " 5.5^{th} "

= $\frac{5th\ item + 6th\ item}{2}$

= $\frac{18+20}{2}$ = 19 Marks

Mode = Here, the variate value 20 is repeated max times. so, mode is 20 Marks.

Range = H.V − L.V = 30 − 8 = 22 Marks.

Co-efficient of Range = $\frac{H.V - L.V}{H.V + L.V}$

$= \frac{22}{30+8} = \frac{22}{38} = 0.5789$

Standard Deviation (S.D) $= \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2} \times h$

$= \sqrt{\frac{402}{10} - \left(\frac{4}{10}\right)^2} \times 1$

$= \sqrt{40.20 - 0.16} \times 1 = \sqrt{40.04}$

$= 6.327$ units.

$$\text{variance} = (S.D)^2$$
$$= (\sqrt{40.04})^2 = 40.04$$

$$C.V = \frac{S.D}{Mean} \times 100$$
$$= \frac{6.327}{18.40} \times 100 = 34.38\%$$

$$\text{co-efficient of } S.D = \frac{S.D}{Mean}$$
$$= \frac{6.327}{18.40} = 0.3438$$

Karl Pearson's coefficient of skewness (Skp)

$$= \frac{Mean - Mode}{S.D} = \frac{18.40 - 20}{6.327} = -0.252$$

Comment: Skp = −0.252 indicates that the distribution is negatively skewed i.e, the distribution has longer tail towards the lower values of variate.

(b) Weight (Kg): 40, 56, 48, 60, 70, 50, 56, 66, 45.

Soln

Working Table

x	$d = x - a/h$	d^2
40	−16	256
45	−11	121
48	−8	64
50	−6	36
56	0	0
56	0	0
60	4	16
66	10	100
70	14	196
N=9	$\sum d = -13$	$\sum d^2 = 789$

Here, $a=56$ $h=1$

Now, $A.M(\bar{x}) = a + \frac{\Sigma d}{N} \times h$

$= 56 - \frac{13}{9} \times 1 = 54.56$ Kgs.

Median = Value of $\left(\frac{N+1}{2}\right)^{th}$ obs

= " " $\left(\frac{9+1}{2}\right)^{th}$ "

= " " 5^{th} "

= 56 Kgs.

Mode = Here the variate value 56 is repeated maximum times so mode is 56 Kgs

Range = H.V − L.V = 70 − 40 = 30 Kgs

co-efficient of Range $= \frac{H.V-L.V}{H.V+L.V} = \frac{30}{70+40} = \frac{30}{110} = 0.2727$

Standard deviation (S.D) $= \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2} \times h$

$= \sqrt{\frac{789}{9} - \left(\frac{-13}{9}\right)^2} \times 1$

$= \sqrt{87.67 - 2.086} = \sqrt{85.584} = 9.251$ Kgs

Variance $= (S.D)^2 = (\sqrt{85.584}) = 85.584$

$C.V = \frac{S.D}{Mean} \times 100 = \frac{9.251}{54.56} \times 100 = 16.95\%$

co-efficient of $S.D = \frac{S.D}{Mean} = \frac{9.251}{54.56} = 0.1695$

Karl Pearson's co-efficient of skewness,

$S_{kp} = \frac{Mean - Mode}{S.D} = \frac{54.56 - 56}{9.251} = -0.156$

Comment : $S_{KP} = -0.156$ indicates that the distribution is negatively negative skewed i.e, the distribution has longer tail towards the lower values of variate.

© Mid value : 5.5 10.5 15.5 20.5 25.5 30.5
Frequency : 2 4 5 4 3 2
(Discrete FD or Ungrouped FD)

Soln

Working Table

Mid value (x)	Frequency (f)	$d = \frac{x-a}{h}$	d^2	fd	$fd^2 = f \times d^2$	c.f
5.5	2	$\frac{5.5-15.5}{5} = -2$	4	-4	8	2
10.5	4	-1	1	-4	4	6
15.5	5	0	0	0	0	11
20.5	4	1	1	4	4	15
25.5	3	2	4	6	12	18
30.5	2	3	9	6	18	20
	N=20	$\Sigma d = 3$	$\Sigma d^2 = 19$	$\Sigma fd = 8$	$\Sigma fd^2 = 46$	

Here, a = 15.5
h = 5

Now, Mean, $(\bar{x}) = a + \frac{\Sigma fd}{N} \times h$
$= 15.5 + \frac{8}{20} \times 5 = 17.5$ units.

Median = Value of $\left(\frac{N+1}{2}\right)^{th}$ obs.
= " " $\left(\frac{20+1}{2}\right)^{th}$ "
= " " 10.5^{th} "
= 15.5 units.

Mode = Here, the variate value 15.5 has highest number of frequency. So, mode = 15.5 units.

Range = H.V - L.V = 30.5 - 5.5
= 25 units.

Co-efficient of Range $= \frac{H.V - L.V}{H.V + L.V}$

$= \frac{30.5 - 5.5}{30.5 + 5.5} = \frac{25}{36} = 0.6944$

$SD(\sigma_x) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times h$

$= \sqrt{\frac{46}{20} - \left(\frac{8}{20}\right)^2} \times 5$

$= \sqrt{2.30 - 0.16} \times 5 = 1.4628 \times 5 = 7.3143$ units.

Variance $(\sigma_x^2) = (S.D)^2 = (\sqrt{2.14} \times 5)^2 = 53.5$ sq units.

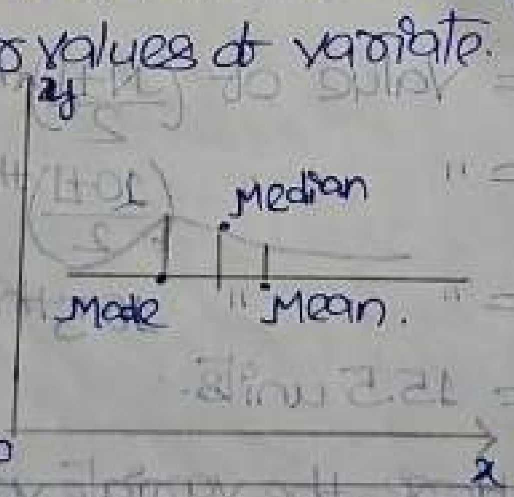
$C.V = \frac{S.D}{Mean} \times 100 = \frac{7.3143}{17.50} \times 100 = 41.796\%$

Co-efficient of S.D $= \frac{S.D}{Mean} = 0.41796.$

Karl Pearson's co-efficient of skewness (S_{KP})

$= \frac{Mean - Mode}{SD} = \frac{17.50 - 15.50}{7.31} = 0.2736$

Comment : $S_{KP} = 0.2736$ indicates that the distribution is positively skewed i.e, the curve has a longer tail towards the higher values of variate.



(d) Central Value : 500 1500 2500 3500 4500 5500 6500

Frequency : 10 12 15 13 9 7 4

Soln

Working Table

x	f	$d=\frac{x-a}{h}$	d^2	fd	fd^2	cf
500	10	$500-3500/1000=-3$	9	-30	90	10
1500	12	-2	4	-24	48	22
2500	15	-1	1	-15	15	37
3500	13	0	0	0	0	50
4500	9	1	1	9	9	59
5500	7	2	4	14	28	66
6500	4	3	9	12	36	70
	$N=70$	$\Sigma d=0$	$\Sigma d^2=28$	$\Sigma fd=-34$	$\Sigma fd^2=226$	

Here $a=3500$, $h=1000$.

Now,

$$\text{Mean}(\bar{x}) = a+\frac{\Sigma fd}{N}\times h$$

$$=3500+\left(\frac{-34}{70}\right)\times 1000 = 3500-485.71$$

$$=3014.29 \text{ units}$$

$$\text{Median} = \text{Value of } \left(\frac{N+1}{2}\right)^{th} \text{ obs.}$$

$$= \text{''} \quad \text{''} \left(\frac{70+1}{2}\right)^{th} \text{''}$$

$$= \text{''} \quad \text{''} \quad 35.5^{th} \text{''}$$

$$=2500 \text{ units}$$

Mode = Here, the variate value 2500 has highest no. of frequency. So, mode = 2500 units

Range = H.V − L.V = 6500 − 500 = 6000 units

$$\text{coefficient of Range} = \frac{H.V-L.V}{H.V+L.V} = \frac{6000}{6500+500} = \frac{6000}{7000}$$

$$=0.8571$$

$$S.D(\sigma_x) = \sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}\times h$$

$$=\sqrt{\frac{226}{70}-\left(\frac{-34}{70}\right)^2}\times 1000$$

$$=\sqrt{3.228-0.235}\times 1000 = \sqrt{2.992}\times 1000$$

$$= 1729.739 \text{ units}$$

$$\text{Variance } (\sigma_x^2) = (S.D)^2 = (1729.739)^2 = 2991997.01$$

$$C.V = \frac{S.D}{Mean}\times 100 = \frac{1729.739}{3014.29}\times 100 = 57.38\%$$

$$\text{Coefficient of } S.D = \frac{S.D}{Mean} = \cancel{0.4179}\ 0.5738$$

Karl Pearson's Co-efficient of Skewness (S_{KP})

$$=\frac{Mean-Mode}{S.D} = \frac{3014.29-2500}{1729.739} = 0.2973$$

Comment: $S_{KP} = \cancel{0.2736}\ 0.2973$ indicates that the distribution is positively skewed i.e, the curve has a longer tail towards the higher values of variate.

Dt: 25/2/16

3. CI: 0-20 0-40 0-60 0-80 0-100
f: 2 5 10 12 20

'OR'

Variate value	less than 20	less than 40	less than 60	less than 80	less than 100
f	2	5	10	12	20

Soln

Working Table

C.I	Frequency (f)	Mid Values (x)	$d=\frac{x-a}{h}$	d^2	fd	fd^2	cf
0-20	2	$\frac{0+20}{2}=10$	-2	4	-4	8	2
20-40	5-2=3	30	-1	1	-3	3	5

40-60	10-5=5	50	0	0	0	0	10
60-80	12-10=2	70	1	1	2	2	12
80-100	20-12 =8	90	2	4	16	32	20
	N=20		Σd=0		Σfd =11	Σfd² =45	

Here, a=50 and N=20

Now Mean $(\bar{x}) = a + \frac{\Sigma fd}{N} \times h$

$= 50 + \frac{11}{20} \times 20 = 61$ units

Median : Here, $\frac{N}{2} = \frac{20}{2} = 10$

Since c.f just greater than 10 is 12. So, median class is 60-80.

we know,

$\therefore$ Median $= L + \frac{N/2 - fc}{f} \times h$

$= 60 + \frac{20/2 - 10}{2} \times 20$

$= 60 + 0 = 60$ units

Here,
l = lower limit of median class
h = length of median class
f = frequency of median class

Mode : Here the highest frequency is '8'. So, modal class is 80-100

we know, Mode $= L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 80 + \frac{8-2}{2\times 8 - 2 - 0} \times 20$

$= 80 + 8.57 = 88.57$ units

Here, f_2 is absent so, f_2 is 0.

Range = H.V − L.V = 100 − 0 = 100 units

co-efficient of Range $= \frac{H.V - L.V}{H.V + L.V}$

$= \frac{100-0}{100+0} = 1.$

$$S.D(\sigma_x) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times h$$

$$= \sqrt{\frac{45}{20} - \left(\frac{11}{20}\right)^2} \times 20$$

$$= \sqrt{2.25 - 0.3025} \times 20 = 27.91057 \text{ units}$$

Variance $(\sigma_x^2) = (27.91)^2 = 778.9681$ sq units

co-efficient of S.D $= \frac{S.D}{Mean}$

$$= \frac{27.91057}{61} = 0.4575$$

$$C.V = \frac{S.D}{Mean} \times 100 = 0.4575 \times 100 = 45.75\%$$

Karl Pearson's co-efficient of Skewness (S_{KP})

$$= \frac{Mean - Mode}{S.D} = \frac{61 - 88.57}{27.91057} = -0.98$$

comment: $S_{KP} = -0.98$ indicates that the distribution is negatively skewed i.e. the curve has a longer tail towards the lower values of variable.

(4) Given data is

	Team A	Team B
No of observations →	20	30
Mean →	40	50
Median →	43	56
S.D →	7	11

i) Which distribution is more skewed?

ii) " " " " variable/unstable/ non-uniform/inconsistent?

iii) Find the combined mean if all observations are taken into A/C?

iv) Find the modal values of both the teams.

Note: we have to ignore the minus sign to see which one is more skewed eg → 2.8, -3.2. -3.2 is more skewed. In short No. higher will be more skewed and sign has no value

Soln

i) We know,

$$Skp = \frac{3(\text{Mean}-\text{Median})}{S.D}$$

For Team A,

$$Skp = \frac{3(40-43)}{7} = -1.2857$$

For Team B,

$$Skp = \frac{3(50-56)}{11} = -1.6363$$

From above, it is clear that Team B is more skewed than Team A.

ii) We know,

$$C.V = \frac{S.D}{\text{Mean}} \times 100$$

For Team A,

$$C.V = \frac{7}{40} \times 100$$
$$= 17.5\%$$

For Team B,

$$C.V = \frac{11}{50} \times 100$$
$$= 22\%$$

From above, it is clear that Team B is more inconsistent.

iii) We know,

Combined Mean, $\bar{x}_{12} = \dfrac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$

$= \dfrac{40 \times 20 + 50 \times 30}{20+30} = \dfrac{2300}{50} = 46$ units

iv) We know,

Mode = 3 Median − 2 Mean

For Team A,

Mode = $3 \times 43 - 2 \times 40 = 49$ units.

For Team B,

Mode = $3 \times 56 - 2 \times 50 = 68$ units.

⑤ Runs scored by Sachin and Dhoni in 5 innings are-

Sachin : 70 60 54 62 65

Dhoni : 80 50 45 65 70

i) Which cricketer scored higher average runs.

ii) " " is more consistent/stable/uniform/non-variable?

iii) Which player would you prefer?

Soln Let 'x' denotes runs scored by Sachin

" 'y' " " " " Dhoni

Working Table

x	$d = \frac{x-a}{h}$	d^2	y	$D = \frac{y-b}{H}$	D^2
54	$\frac{54-62}{1} = -8$	64	45	$45-65/1 = -20$	400
60	−2	4	50	−15	225
62	0	0	65	0	0
65	3	9	70	5	25
70	8	64	80	15	225
	$\Sigma d = 1$	$\Sigma d^2 = 141$		$\Sigma D = -15$	$\Sigma D^2 =$ 875

Here $a = 62$, $h = 1$

Here, $a = 65$

we know,

$Mean(\bar{x}) = a + h \times \frac{\Sigma d}{n}$

$= 62 + 1 \times \frac{1}{5}$

$= 62.20$ runs

we know,

$Mean(\bar{y}) = b + H \times \frac{\Sigma D}{n}$

$= 65 + 1 \times \frac{-15}{5}$

$\bar{y} = 62$ runs

$S.D(\sigma x) = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2} \times h$

$= \sqrt{\frac{141}{5} - \left(\frac{1}{5}\right)^2} \times 1$

$= \sqrt{28.2 - 0.04}$

$= 5.30$

$S.D(\sigma y) = \sqrt{\frac{\Sigma D^2}{n} - \left(\frac{\Sigma D}{N}\right)^2} \times h$

$= \sqrt{\frac{875}{5} - \left(\frac{-15}{5}\right)^2} \times 1$

$= \sqrt{\cancel{405} - 9\ 175 - 9}$

$= 12.8840$

$C.V = \frac{S.D}{Mean} \times 100$

$= \frac{5.30}{62.20} \times 100$

$= 8.52\%$

$C.V = \frac{S.D}{Mean} \times 100$

$= \frac{12.88}{62} \times 100$

$= 20.77\%$

i) Clearly,
Sachin scored higher average score than Dhoni.

ii) Also,
C.V of Sachin < C.V. of Dhoni

∴ Sachin is more consistent than Dhoni in scoring runs.

iii) We would prefer Sachin

⑥ The average marks obtained by certain boys and girls are respectively 60 and ~~88~~ 68. The avg. of all students is 66 Marks.

i) Find the ratio of boys and girls.

ii) Find the P.C. of boys and girls.

Soln

Given,

$\bar{x}_1 = 60$

$\bar{x}_2 = 68$

$\bar{x}_{12} = 66$

We know, $\bar{x}_{12} = \frac{x_1 n_1 + x_2 n_2}{n_1 + n_2}$

$\Rightarrow 66 = \frac{n_1 \times 60 + n_2 \times 68}{n_1 + n_2}$

$\Rightarrow 66n_1 - 60n_1 = 68n_2 - 66n_2$

$\Rightarrow 3n_1 = n_2$

$\Rightarrow n_1 : n_2 = 1:3$

i) Ratio of boys and girls is 1:3.

ii) P.C. of boys $= \frac{1}{1+3} \times 100 = \frac{1}{4} \times 100 = 25\%$

P.C. of girls $= \frac{3}{1+3} \times 100 = \frac{3}{4} \times 100 = 75\%$

Dt: 12/2/16

(7) Given data is -

	Firm A	Firm B
No. of workers →	200	250
Avg daily wage (mean) →	400	310
Variance daily wage →	144	121

(a) Which firm pays out larger daily wage?

(b) " " is more consistent in paying daily wage

(c) Find the combined avg. daily wage.

Soln

i) Firm A Pays $= n_1 \times \bar{x}_1 = 200 \times 400 = ₹80000$

Firm B Pays $= n_2 \times \bar{x}_2 = 250 \times 310 = ₹77500$

From above it is clear that firm A pays larger daily wage.

ii) We know, $CV = \frac{S.D}{Mean} \times 100$ | $S.D = \sqrt{Variance}$

Firm A $= \frac{\sqrt{144}}{400} \times 100 = \frac{12}{4} 3 = 3\%$

Firm B $= \frac{\sqrt{121}}{310} \times 100 = \frac{11\cancel{0}}{31\cancel{0}} \times 1\cancel{0}\cancel{0} = 3.548\%$

Since CV of firm A < CV of firm B

∴ Firm A is more consistent in paying daily wage.

iii) We know $\bar{x}_{12} = \frac{n_1 \times \bar{x}_1 + n_2 \times \bar{x}_2}{n_1 + n_2}$

$$= \frac{80000 + 77500}{200 + 250} = \frac{157500}{450} = ₹350$$

8) In a symmetrical distribution, the sum of lower and upper quartiles and upper quartiles is 40 and their difference is 30. Find

i) Q_2 or Second Quartile or Median.

ii) Find Bowley's co-efficient of skewness.

Soln
i) Given, $Q_3 + Q_1 = 40 \rightarrow (I)$

$Q_3 - Q_1 = 30 \rightarrow (II)$

(I)+(II)

$Q_3 + Q_1 + Q_3 - Q_1 = 40 + 30$

$\Rightarrow 2Q_3 = 70$

$\Rightarrow Q_3 = 35$

From (I)

$Q_3 + Q_1 = 40$

$\Rightarrow 35 + Q_1 = 40$

$\Rightarrow Q_1 = 5$

Since the distribution is symmetrical

$\therefore Q_3 - Q_2 = Q_2 - Q_1$

$\Rightarrow 35 - Q_2 = Q_2 - 5$

$\Rightarrow 2Q_2 = 40$

$\Rightarrow Q_2 = 20$

$\therefore Q_2$ or median = 20 units.

ii) Bowley's co-efficient of skewness (S_B)

$$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{(35-20) - (20-5)}{\cancel{30=}(35-20) + (20-5)} = \frac{15-15}{15+15} = \frac{0}{30} = 0$$

Find Quartile Deviation (Q.D) or Semi Inter-Quartile Range, Inter-Quartile Range coefficient of Q.D and Bowley's co-efficient of skewness.
[Imp Note: Q_2 is also Known as Median]

(a) Marks: 10, 15, 5, 28, 19, 24, 35, 40

Soln

Arrangement of Data-

Marks: 5, 10, 15, 19, 24, 28, 35, 40

Q_1 = Value of $\left(\frac{N+1}{4}\right)$th item.

= " " $\left(\frac{8+1}{4}\right)$th "

= " " 2.25th "

= 2nd item + 0.25 × (3rd item − 2nd item)

= 10 + 0.25 × (15 − 10) = 10 + 1.25 = 11.25 Marks

Q_2 = Value of $2\left(\frac{N+1}{4}\right)$th item.

= " " (2×2.25)th "

= " " 4.5th item.

$= \frac{19+24}{2} = 21.5$ Marks.

Q_3 = Value of $3\left(\frac{N+1}{4}\right)$th item

= " " $3 \times (2.25)$th "

= " " 6.75th item.

= 6th item + 0.75 × (7th item − 6th item)

= 28 + 0.75(35 − 28) = 28 + 5.25 = 33.25 Marks.

Now,

Q.D or Semi Inter Quartile Range

$= \frac{Q_3 - Q_1}{2} = \frac{33.25 - 11.25}{2} = 11$ Marks

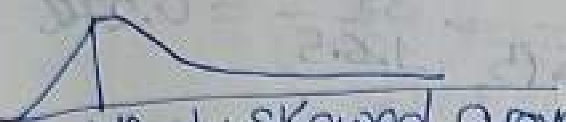
Interquartile Range $= Q_3 - Q_1 = 22$ Marks.

Bowley's Co-efficient of Skewness (SB)

$= (Q_3 - Q_2) - (Q_2 - Q_1) / (Q_3 - Q_2) + (Q_2 - Q_1)$

$$= \frac{(33.25-21.5)-(21.5-11.25)}{(33.25-21.5)+(21.5-11.25)} = \frac{1.5}{22} = 0.06818$$

Comment: $S_B = 0.0682$ indicates that the distribution is positively skewed i.e, the curve has a longer tail towards the higher values of variate.



Positively Skewed curve.

b) Weight (Kg): 50, 70, 65, 42, 86, 90, 75, 79, 35, 47.

Soln Arrangement of Data

Weight (Kg): 35, 42, 47, 50, 65, 70, 75, 79, 86, 90

Q_1 = Value of $\left(\frac{N+1}{4}\right)$th item.

$= \text{" " } \left(\frac{10+1}{4}\right)$th "

= " " 2.75th "

= 2nd item + 0.75 × (3rd item − 2nd item)

$= 42 + 0.75 \times (47-42) = 42 + 3.75 = 45.75$ Kg

Q_2 = Value of $2\left(\frac{N+1}{4}\right)$th item.

= " " (2×2.75)th "

= " " 5.5th "

$= \cancel{\frac{86+90}{2} = 88}\ \frac{65+70}{2} = 67.50$ Kg.

Q_3 = Value of $3\left(\frac{N+1}{4}\right)$th item.

= " " (3×2.75)th "

= " " 8.25th "

= 8th item + 0.25 × (9th item − 8th item)

$= 79 + 0.25 \times (86-79)$

$= 79 + 1.75 = 80.75$ Kg

$\therefore$ QD $= \frac{Q_3 - Q_1}{2} = \frac{80.75 - 45.75}{2} = 17.50$ Kg

Inter Quartile Range $= Q_3 - Q_1 = 35$ Kg

coefficient of Range $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$ QD

$= \frac{35}{80.75 + 45.75} = \frac{35}{126.5} = 0.2766$

Bowley's coefficient of Skewness (S_B)

$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$

$= \frac{(80.75 - 67.50) - (67.50 - 45.75)}{(80.75 - 67.50) + (67.50 - 45.75)}$

$= \frac{13.25 - 21.75}{13.25 + 21.75} = \frac{8.5}{35} = -0.2428$

comment : $S_B = -0.2428$ indicates that the distribution is negatively skewed i.e, the curve has a longer tail towards the lower values of variate.

Dt 27/12/16

ⓒ

x :	20	40	60	80	100	120
f :	4	7	10	8	4	3

Soln

Working Table

x	frequency (f)	Cumulative frequency (cf)
20	4	4
40	7	11
60	10	21
80	8	29
100	4	33
120	3	36
	N=36	

Now, Q_1 = Value of $\left(\frac{N+1}{4}\right)^{th}$ item
= " " $\left(\frac{36+1}{4}\right)^{th}$ "
= " " 9.25^{th} "
= 40 units

Q_2 = Value of $2\left(\frac{N+1}{4}\right)^{th}$ item
= " " $(2\times9.25)^{th}$ "
= " " 18.50^{th} "
= 60 units.

Q_3 = Value of $3\left(\frac{N+1}{4}\right)^{th}$ item.
= " " $(3\times9.25)^{th}$ "
= " " 27.75^{th} "
= 80 units.

$\therefore Q.D = \frac{Q_3-Q_1}{2} = \frac{80-40}{2} = \frac{40}{2} = 20$ units.

Inter-Quartile Range $= Q_3 - Q_1 = 80-40 = 40$ units.

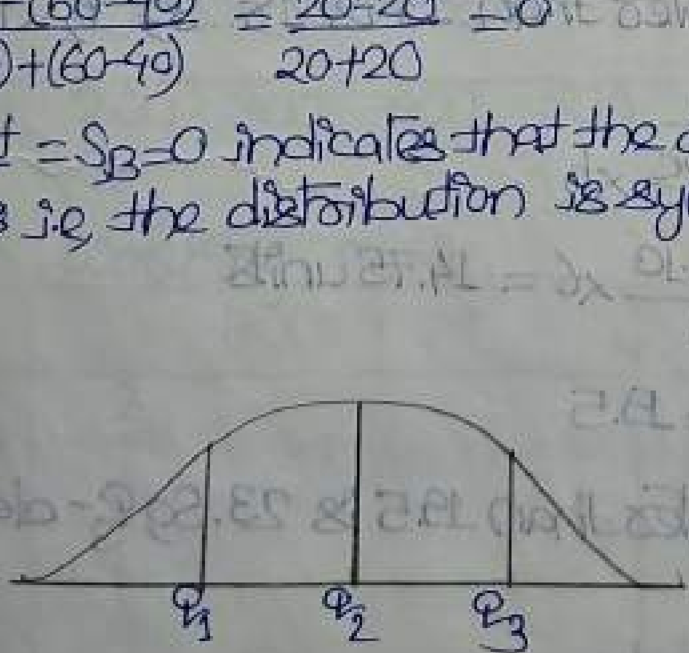
coefficient of $Q.D = \frac{Q_3-Q_1}{Q_3+Q_1} = \frac{40}{80+40} = \frac{40}{120} = 0.333$

Bowley's coefficient of skewness (S_B)

$$= \frac{(Q_3-Q_2)-(Q_2-Q_1)}{(Q_3-Q_2)+(Q_2-Q_1)}$$

$$= \frac{(80-60)-(60-40)}{(80-60)+(60-40)} = \frac{20-20}{20+20} = 0$$

Comment = $S_B=0$ indicates that the distribution has no skewness i.e, the distribution is symmetrical about Q_2



$S_B = 0$

① C.I : 1-6 7-12 13-18 19-24 25-30 31-36
f : 6 4 8 5 1 2

Soln

Working Table

C.I.	C.B.	f	cf
1-6	0.5-6.5	6	6
7-12	6.5-12.5	4	10
13-18	12.5-18.5	8	18
19-24	18.5-24.5	5	23
25-30	24.5-30.5	1	24
31-36	30.5-36.5	2	26
		N=26	

Here, $\frac{N}{4} = \frac{26}{4} = 6.5$

Since cf just greater than 6.5 is 10. So, Q_1 class is 6.5-12.5

$\therefore Q_1 = L + \frac{N/4 - fc}{f} \times h$

$= 6.5 + \frac{6.5 - 6}{4} \times 6 = 7.25$ units.

Again, $\frac{2N}{4} = 2 \times 6.5 = 13$

Since cf just greater than 13 is 18. So, Q_2 class is 12.5-18.5

$\therefore Q_2 = L + \frac{2N/4 - fc}{f} \times h$

$= 12.5 + \frac{13 - 10}{8} \times 6 = 14.75$ units

And, $\frac{3N}{2} = 6.5 \times 3 = 19.5$

Since c.f just greater than 19.5 is 23. So Q_3 class is 18.5-24.5

$\therefore Q_3 = L + \frac{3N/4 - fc}{f} \times h$

$= 18.5 + \frac{19.5-18}{5} \times 6 = 20.30$ units

Now, $Q.D = \frac{Q_3 - Q_1}{2} = \frac{20.30 - 7.25}{2} = 6.525$ units

Inter Quartile Range $= Q_3 - Q_1 = 13.05$ units

coefficient of $Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

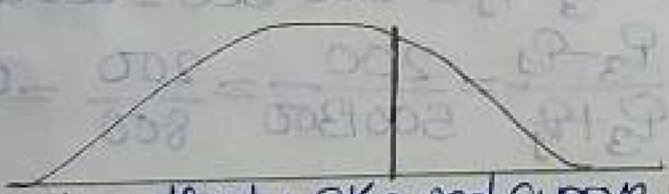
$= \frac{20.30-7.25}{20.30+7.25} = \frac{13.05}{27.55} = 0.4736$

Bowley's coefficient of skewness (S_B)

$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$

$= \frac{(20.30-14.75)-(14.75-7.25)}{(20.30-14.75)+(14.75-7.25)} = \frac{-1.95}{13.05} = -0.1494$

Comment: $S_B = 0.1494$ indicates that the distribution is negatively skewed i.e. the distribution has a longer tail towards the lower values of variate.



Negatively Skewed Curve

(c) x : 100 200 300 400 500 600 700
f : 2 3 4 5 3 2 1

Soln Working Table

x	f	cf
100	2	2
200	3	5
300	4	9
400	5	14
500	3	17

600	2	19
700	1	20
	$N=20$	

Now, Q_1 = Value of $\left(\frac{N+1}{4}\right)^{th}$ item.
$= " \quad " \left(\frac{20+1}{4}\right)^{th}$ "
$= " \quad " \; 5.25^{th}$ "
$= 300$ units

Q_2 = Value of $2\left(\frac{N+1}{4}\right)^{th}$ item
$= " \quad " (2\times5.25)^{th}$ "
$= " \quad " \; 10.50^{th}$ "
$= 400$ units.

Q_3 = Value of $3\left(\frac{N+1}{4}\right)^{th}$ "
$= " \quad " (3\times5.25)^{th}$ "
$= " \quad " \; 15.75^{th}$ "
$= 500$ units.

Now, $QD = \frac{Q_3-Q_1}{2} = \frac{500-300}{2} = \frac{200}{2} = 100$ units

Interquartile Range $= Q_3 - Q_1 = 500-300 = 200$ units.

Co-efficient of Q.D $= \frac{Q_3-Q_1}{Q_3+Q_1} = \frac{200}{500+300} = \frac{200}{800} = 0.25$

Bowley's co-efficient of skewness (S_B)

$$= \frac{(Q_3-Q_2)-(Q_2-Q_1)}{(Q_3-Q_2)+(Q_2-Q_1)}$$

$$= \frac{(500-400)-(400-300)}{(500-400)+(400-300)} = \frac{100-100}{100+100} = 0$$

Comment: $S_B = 0$ indicates that the distribution has no skewness i.e, the distribution is symmetrical about Q_2.

(6) C.I: 0-50 50-150 150-220 220-300 300-340 340-400
f: 20 30 45 25 10 15

Soln

Working Table

CI	f	cf
0-50	20	20
50-150	30	50
150-220	45	95
220-300	25	120
300-340	10	130
340-400	15	145
	$N=145$	

Now $\frac{N}{4}=\frac{145}{4}=36.25$

Here, c.f just greater than 36.25 is 50. So, Q_1-class is 50-150.

$\therefore Q_1 = L+\frac{N/4-fc}{f}\times h$

$=50+\frac{36.25-20}{30}\times 100 = 104.17$ units

Again, $\frac{2N}{4}=2\times 36.25=72.50$

Here c.f just greater than 72.50 is 95 so Q_2 class is 150-220

$\therefore Q_2 = L+\frac{2N/4-fc}{f}\times h$

$=150+\frac{72.50-50}{45}\times 70 = 185$ units

Also, $\frac{3N}{4}=3\times 36.25=108.75$

Here, cf just greater than 108.75 is 120. So, Q_3-class is 220-300.

$\therefore Q_3 = L+\frac{3N/4-fc}{f}\times h$

$=220+\frac{108.75-95}{25}\times 80 = 264$ units

Now, $Q.D = Q_3 - Q_1/2 = \frac{264-104.17}{2} = 79.915$ units

Inter Quartile Range $= Q_3 - Q_1 = 264-104.17 = 159.83$ units

coefficient of $Q.D = \frac{Q_3-Q_1}{Q_3+Q_1} = \frac{264-104.17}{264+104.17} = \frac{79.915}{368.17}$
$= 0.21706$

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(10) If $Skp = 0.40$, $C.V = 20\%$, Median $= 60$. Find
a) Mean (b) S.D (c) Variance (d) Mode.

Soln Given, Median $= 60$

$C.V = 20\%$

$Skp = 0.4$

$\Rightarrow \frac{3(\text{Mean}-\text{Median})}{S.D} = 0.40$

$\Rightarrow 3(\text{Mean}-60) = 0.40.S.D$

$\Rightarrow 3\text{Mean} - 180 = 0.40 S.D$

$\Rightarrow 7.50\text{ Mean} - 450 = S.D \longrightarrow (1)$

Again, $CV = \frac{S.D}{\text{Mean}} \times 100$

$\Rightarrow 20 = \frac{(7.50\text{ Mean} - 450)100}{\text{Mean}}$

$\Rightarrow 20\text{ Mean} = 750\text{ Mean} - 45000$

a) $\Rightarrow \text{Mean} = \frac{45000}{730} = 61.64$ units.

b) $S.D = 7.50 \times 61.64 - 450 = 462.30 - 450 = 12.30$ units.

c) Variance $= (S.D)^2 = (12.30)^2 = 151.2929$ units.

d) Again, Mode $= 3\text{ Median} - 2\text{ Mean}$

$= 3\times 60 - 2\times 61.64 = 56.72$ units

and Dt: 19/2/16

(11) Find Mean Deviation (M.D) about mean of coefficient of M.D about mean from the following data:

(a) x: 10, 15 18, 6, 20. Hence find Q.D and S.D

Soln

Working Table

x	$d = \frac{x-a}{h}$	$\lvert x-\bar{x}\rvert$
6	-9	7.80
10	-5	3.80
15	0	1.20
18	3	4.20
20	5	6.20
	$\Sigma d = -6$	$\Sigma\lvert x-\bar{x}\rvert = 23.20$

Here, $a = 15$, $H = 1$, $N = 5$

Now, Mean $(\bar{x}) = a + \frac{\Sigma d}{N} \times h$

$= 15 - \frac{6}{5}\times 1 = 15 - 1.20 = 13.80$ units.

Again, M.D about Mean $= \frac{\Sigma\lvert x-\bar{x}\rvert}{N} = \frac{23.20}{5} = 4.64$ units.

co-efficient of Mean Deviation about Mean

$= \frac{M.D}{Mean} = \frac{4.64}{13.80} = 0.3362$

We know,

$6Q.D = 5M.D = 4S.D$

$\Rightarrow 6Q.D = 5\times 4.64 = 4S.D$

$\therefore Q.D = \frac{5\times 4.64}{6} = 3.867$

$S.D = \frac{5\times 4.64}{4} = 5.80$

(b) x : 10 20 30 40 50 60
f : 2 4 5 3 2 1

Soln

Working Table

x	f	$d=\frac{x-a}{h}$	fd	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10	2	-2	-4	21.2	42.4
20	4	-1	-4	11.2	44.8
30	5	0	0	1.2	6
40	3	1	3	8.8	26.4
50	2	2	4	18.8	37.6
60	1	3	3	28.8	28.8
			$\Sigma fd=2$		$\Sigma f\lvert x-\bar{x}\rvert=186$

Here $N=17$, $a=30$, $H=10$

Now, Mean$(\bar{x}) = a+\frac{\Sigma fd}{N}\times h$

$$= 30+\frac{2}{17}\times 10 = 30+\frac{20}{17} = 31.20 \text{ units.}$$

Mean Deviation about Mean $= \frac{\Sigma f\lvert x-\bar{x}\rvert}{N} = \frac{186}{17} = 10.9411$ units.

coefficient of M.D about Mean $= \frac{M.D}{Mean} = \frac{10.9411}{31.20} = 0.3506$

we know,

$6Q.D = 5M.D = 4S.D$

$\Rightarrow 6Q.D = 5\times 10.9411 = 4S.D$

$\therefore Q.D = \frac{5\times 10.9411}{6} = 9.1176$ units.

$S.D = \frac{5\times 10.9411}{4} = 13.6763$ units.

(c) C.I: 1-4 5-8 9-12 13-16 17-20
f : 6 8 10 4 2

Soln

Working Table

CI	f	Mid Value (x)	$d=\frac{x-a}{h}$	fd	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
1-4	6	$5/2=2.5$	-2	-12	6.9	41.4
5-8	8	6.5	-1	-8	2.9	23.2
9-12	10	10.5	0	0	1.1	11
13-16	4	14.5	1	4	5.1	20.4
17-20	2	18.5	2	8	9.1	18.2
	N=30			$\Sigma fd=-8$		$\Sigma f\lvert x-\bar{x}\rvert=114.2$

Here $a=10.5$, $H=4$.

Now, Mean, $\bar{x}=a+\frac{\Sigma fd}{N}\times h$

$=10.5-\frac{8}{30}\times 4=10.5-1.067=9.40$ units.

M.D about Mean $=\frac{\Sigma f\lvert x-\bar{x}\rvert}{N}=\frac{114.20}{30}=3.80$ units

co-efficient of M.D $=\frac{M.D}{Mean}=\frac{3.80}{9.40}=0.40496$

We know,

$6Q.D=5M.D=4S.D$

$\Rightarrow 6Q.D=5\times3.80=4.S.D$

$\therefore Q.D=\frac{5\times3.80}{6}=3.167$ units.

$S.D=\frac{5\times3.80}{4}=4.75$ units.

(12) Find S.D-

(a) 4 and 10

Soln

$S.D=\frac{1}{2}(10-4)=\frac{1}{2}\times6=3$ units.

(b) 3,6,3,6,3,6

Soln $S.D=\frac{1}{2}(6-3)=\frac{1}{2}\times3=1.5$ units.

(c) 1,2,3,4,5,6,7. [2013] (3)

Soln $S.D = \sqrt{n^2-1/12}$

Here, $n=7$. $\therefore S.D = \sqrt{7^2-1/12} = \sqrt{\frac{49-1}{12}} = \sqrt{4} = 2$ units.

(d) 1,2, - - 9

Soln Here, $n=9$.

$\therefore S.D = \sqrt{n^2-1/12} = \sqrt{9^2-1/12} = \sqrt{80/12} = 3.46$ units.

(13) If S.D of 'n' natural numbers is $\frac{\sqrt{133}}{2}$. Find n.

Soln We know,

S.D of 'n' natural numbers $= \sqrt{\frac{n^2-1}{12}}$

$\Rightarrow \frac{\sqrt{133}}{2} = \sqrt{\frac{n^2-1}{12}}$

$\Rightarrow \frac{133}{4} = \frac{n^2-1}{12}$ (squaring both sides)

$\Rightarrow 399+1 = n^2$

$\Rightarrow n^2 = (20)^2$

$\Rightarrow n = 20$.

Dt: 24/2/16

(14) Find A.M, G.M, H.M of

(a) 3,9,27 (b) 4,8,16

Also show that-

i) AM > GM > HM

ii) $(GM)^2 = AM \times HM$

Soln

(a) Given data is 3,9,27

$\therefore AM = \frac{3+9+27}{3} = \frac{39}{3} = 13$ units.

$GM = \sqrt[3]{3\times9\times27} = \sqrt[3]{3^1\times3^2\times3^3} = 3^{\left(\frac{1+2+3}{3}\right)} = 3^{6/3} = 3^2$

$= 9$ units

$HM = \frac{3}{1/3+1/9+1/27}$

$= \frac{3}{\frac{9+3+1}{27}} = \frac{81}{13} = 6.75$ units.

From above it is clear than $13 > 9 > 6.75$

i) ∴ AM > GM > HM Proved

ii) To proove: $(GM)^2 = AM \times HM$

L.H.S	R.H.S
$(GM)^2$	$AM \times HM$
$= (9)^2$	$= 13 \times \frac{81}{13}$
$= 81$	$= 81$

∴ L.H.S = R.H.S

Hence Proved

⑭ Given data is 4, 8, 16

∴ $AM = \frac{4+8+16}{3} = \frac{28}{3} = 9.33$ units.

$G.M = \sqrt[3]{4 \times 8 \times 16} = \sqrt[3]{2^2 \times 2^3 \times 2^4} = 2^{2+3+4/3} = 2^{9/3} = 2^3 = 8$ units.

$H.M = \frac{3}{\frac{1}{4} + \frac{1}{8} + \frac{1}{16}} = \frac{3}{\frac{4+2+1}{16}} = \frac{3 \times 16}{7} = \frac{48}{7} = 6.86$ units

From above it is clear that $9.33 > 8 > 6.86$

i) ∴ AM > GM > HM Proved

ii) To proove: $(GM)^2 = AM \times HM$

L.H.S	R.H.S
$(GM)^2$	$AM \times GM$
$= (8)^2$	$= \frac{28}{3} \times \frac{48}{7}$
$= 64$	$= 64$

∴ L.H.S = R.H.S

Hence Proved

⑮ Under what condition will AM, GM and HM be equal?

→ If all observations (or variable values) are equal than AM, G.M and H.M will be equal (identical).

(16) Find the two obs. if
i) A.M = 7.5, G.M = 6. Hence find H.M
ii) AM = 7.5, HM = $\frac{20}{3}$ " " GM
iii) GM = 16, HM = 8 " " AM.

$AM = \frac{x+y}{2}$

$GM = \sqrt{xy}$

Soln: Let the two observations be 'x' and 'y'

Given, A.M = 7.5

$\Rightarrow \frac{x+y}{2} = 7.5$

$\Rightarrow x+y = 15 \quad \Rightarrow x = 15-y \rightarrow (1)$

And, G.M = 6

$\Rightarrow \sqrt{xy} = 6$

$\Rightarrow xy = 36$ (squaring)

$\Rightarrow (15-y)y = 36$

$\Rightarrow y^2 - 15y + 36 = 0$

$\Rightarrow (y-12)(y-3) = 0$

Either $y-12=0 \Rightarrow y = 12$, $\therefore x = 15-12 = 3$

or $y-3=0 \Rightarrow y = 3$, $x = 15-3 = 12$

$\Rightarrow$ Reqd observations are (12,3) or and (3,12)

12 and 3

we know, $(GM)^2 = AM \times HM$

$\Rightarrow \frac{6\times 6}{7.5} = H.M$

$\Rightarrow H.M = 4.80$ units

(ii) Let the two observations be 'a' and 'b'

Given, A.M = 7.5 And, H.M = $\frac{20}{3}$

$\Rightarrow \frac{a+b}{2} = 7.5$

$\Rightarrow a+b = 15$

$\Rightarrow a = 15-b \rightarrow (1)$

$\Rightarrow \frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{20}{3}$

$\Rightarrow \frac{2}{\frac{a+b}{ab}} = \frac{20}{3}$

$\Rightarrow \frac{2ab}{15} = \frac{20}{3}$

$\Rightarrow (15-b)b = 50$

$\Rightarrow b^2 - 15b + 50 = 0$

$\Rightarrow (b-10)(b-5) = 0$

Either $b-10=0 \Rightarrow b = 10$, $\therefore a = 15-10 = 5$

or $b-5=0 \Rightarrow b = 5$, $\therefore a = 15-5 = 10$

$\therefore$ Reqd observations are (10, 5) or (5, 10).

we know, $GM = \sqrt{AM \times HM}$ $\qquad (GM)^2 = AM \times HM$

$$= \sqrt{\frac{\cancel{15}\,15}{2} \times \frac{\cancel{20}\,10}{\cancel{3}}} = \sqrt{50} = 7.07 \text{ units}$$

(iii) Let the two observations be 'a' and 'b'

Given, G.M = 10

$\Rightarrow \sqrt{ab} = 10$

$\Rightarrow ab = 100 \rightarrow (i)$

$\Rightarrow a = \frac{100}{b} \rightarrow (ii)$

H.M = 8

$\Rightarrow \frac{2}{\frac{1}{a}+\frac{1}{b}} = 8$

$\Rightarrow \frac{2}{\frac{a+b}{ab}} = 8$

$\Rightarrow \frac{2ab}{a+b} = 8$

$\Rightarrow \frac{2 \times \cancel{100}^{25}}{\frac{100}{b}+b} = \cancel{8}\,4$

$\Rightarrow \frac{25}{100+b^2} \times b = 1$

$\Rightarrow b^2 - 25b + 100 = 0$

$\Rightarrow (b-20)(b-5) = 0$

Either $b-20=0$ $\Rightarrow b = 20$ $\therefore a = \frac{100}{20} = 5$

or $b-5=0$ $\Rightarrow b=5$ $\therefore a = \frac{100}{5} = 20$

$\therefore$ The reqd. observations are (20, 5) or (5, 20)

Now, A.M $= \frac{(G.M)^2}{H.M}$

$$= \frac{\cancel{10} \times \cancel{10}\,5}{\cancel{8}\,\cancel{4}\,2} = 12.50 \text{ units}.$$

(17) A man goes uphill at a speed of 40 Km/hr and returns back/downhill at a speed of 60 Km/hr. Find the man's avg. speed.

Soln Let the distance between starting point and final point be 'x' Km.

we know,

$$\text{Avg speed} = \frac{\text{Total Distance Travelled}}{\text{Total time taken}}$$

$$= \frac{x+x}{x/40 + x/60}$$

$S = D/T$

$\therefore T = D/S$

$$= \frac{2x}{x\left(\frac{3+2}{120}\right)} = \frac{2\times120}{5} = 48\ Km/hr$$

'OR'

Since the distance is fixed while going uphill and returning downhill. So, H.M is the best avg. to determine avg. speed.

we know, $H.M = \frac{2}{1/40 + 1/60} = \frac{2\times120}{5} = 48\ Km/hr$

Dt: 26/2/15

(18) Avg. marks obtained by 60 students are 70. If later on it was discovered that two observations 60 and 50 were taken as 06 and 05 respectively. Find correct Mean.

Soln we know, Mean $(\bar{x}) = \frac{\Sigma x}{n}$

$\therefore$ wrong mean $(\bar{x}) = \frac{\text{wrong } \Sigma x}{n}$

$\Rightarrow 70 \times 60 = \text{wrong } \Sigma x$

$\Rightarrow 4200 = \text{wrong } \Sigma x$

$\because$ 60 and 50 is taken as 06 and 05 respectively

$\therefore$ correct $\Sigma x = 4200 + 60 + 50 - 06 - 05 = 4299$

$\therefore$ correct Mean $= \frac{\text{correct } \Sigma x}{n} = \frac{4299}{60} = 71.65$ Marks

(19) A candidate obtains the following % in an exam - English - 54%, Maths - 70%, Stats - 80%, Eco - 40% and Logic - 60%.

It is agreed to double the weights to marks in eng and maths, triple the weight to the stats.

i) Find the candidate's simple avg marks.

ii) " " " weighted " "

Soln i) candidate's simple avg $(\bar{x})$

$$= \frac{54+70+80+40+60}{5} = \frac{304}{5} = 60.8\%$$

ii) Candidate's weighted avg ($\bar{x}_w$)

$$= \frac{54\times2+70\times2+80\times3+40+60}{2+2+3+1+1}$$

$$= \frac{108+140+240+100}{9} = \frac{588}{9} = 65.33\%$$

(20) Find the missing frequency if Mode = 44 units.

C.I : 10-20 20-30 30-40 40-50 50-60 60-70
f : 5 8 12 x 10 8

Soln since Mode = 44 units.

∴ The modal class is 40-50

We know

$$\text{Mode} = L + \frac{f_1-f_0}{2f_1-f_0-f_2}\times h$$

$$\Rightarrow 44 = 40 + \frac{x-12}{2\times x-12-10}\times 10$$

$$\Rightarrow \frac{(44-40)}{10} = \frac{x-12}{2x-22}$$

$$\Rightarrow 4(2x-22) = 10(x-12)$$

$$\Rightarrow 8x-88 = 10x-120$$

$$\Rightarrow 2x = 32$$

$$\Rightarrow x = 16.$$

∴ The missing frequency is 16.

*(21) Find the missing frequency if Median = 32 units.

C.I : 0-10 10-30 30-60 60-70 70-90
f : 15 25 x 4 10

Soln

Working Table

C.I.	f	C.f
0-10	15	15
10-30	25	40
30-60	x	$40+x$
60-70	4	$44+x$
70-90	10	$54+x$

$N = 54 + x$

Here, Median = 32 units.

∴ Median class is 30-60 units.

We know,

$$\text{Median} = L + \frac{\frac{N}{2} - cf}{f} \times h^{*}$$

[Diff of length in Median class]

$$\Rightarrow 32 = 30 + \frac{\frac{54+x}{2} - 40}{x} \times 30$$

$$\Rightarrow (32-30)x = \left(\frac{54+x-80}{2}\right) \times \cancel{30}\,15$$

$$\Rightarrow 2x = \cancel{26}(x-26)15$$

$$\Rightarrow 2x - 15x = -26 \times 15$$

$$\Rightarrow -13x = -26 \times 15$$

$$\Rightarrow x = 30.$$

∴ The missing frequency is 30.