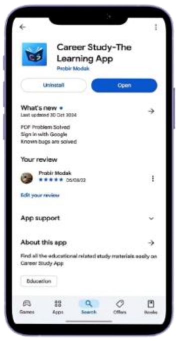
ECODSC – 251

CAREER STUDY
The Learning App

## ABOUT CAREER STUDY:

**Career Study** is a forward-thinking educational platform designed to empower students across India, with a special focus on the Northeast region. Recognized as a **"Startup"** under the Assam Startup Policy 2016 by the Government of Assam and officially registered under the MSME, Government of India.




At Career Study, we believe in creating opportunities, fostering growth, and inspiring success—one student at a time.

## ABOUT THE FOUNDER:

**Probir Modak**, the founder of Career Study, is an educator and entrepreneur from Silcoorie Camp and currently residing in Silchar, Kathal Road, Probir brings over 8 years of teaching experience and a deep passion for empowering students. His journey began at Assam University, Silchar.

Probir's achievements are deeply rooted in the unwavering support of his family, especially his mother (MAA), whose encouragement and guidance have been pivotal in his journey. Armed with a Master's degree in Economics from Assam University, Silchar.

## SUPPORT US:

**PLAY STORE** – CAREER STUDY

**YOUTUBE** – CAREER STUDY

**FACEBOOK** – CAREER STUDY

**WHATSAPP** – CAREER STUDY

**LEARN. GROW. SUCCEED.**

**Semester- IV**

**ECODSC – 251**

**Intermediate Microeconomics**

**Total Credits: 3**

---

**Unit 1: Consumer Behaviour**

Ordinary vs compensated demand curve, derivation of ordinary and compensated demand function, indirect utility, income and substitution effect of normal and inferior goods, application of indifference curve: labour – leisure trade off, cash subsidy vs food transfer, Revealed preference theory.

**Unit 2: Cost and Production Function**

Homogenous and homothetic production function, Cobb-Douglas and C.E.S. production function, elasticity of factor substitution, expansion path, derivation of cost function from production function.

**Unit 3: Supply decision in imperfect competition:**

Price output determination under monopolistic market, collusive vs non-collusive oligopoly, Cournot, Bertrand and Stackelberg's model, Kinked demand curve, concept of collusive oligopoly, introduction to game theory in understanding duopoly market (two-person zero sum game).

**Unit 4: Factor Market:**

Marginal productivity of a factor: marginal physical product, marginal revenue product and value of the marginal product, marginal productivity theory of factor pricing, equilibrium of a firm in the factor market under perfect competition, factor market equilibrium under imperfect competition, exploitation of labour and minimum wage bill.

**Unit 5: Welfare Economics**

Nature of welfare economics, Pigouvian welfare criterion, Pareto optimality criterion, Kaldor-Hicks compensation criterion, Social welfare function.

UNIT – 1

THEORY:

## 1.1 Ordinary vs Compensated Demand Curve

### Ordinary Demand Curve:

- Also known as the **Marshallian demand curve**, it shows the relationship between the price of a good and the quantity demanded, assuming **income and other factors remain constant**.
- Derived from the utility maximization principle, it reflects actual consumer choices based on market prices and budget constraints.
- When the price of a good decreases, the consumer buys more due to **both income effect and substitution effect**.

### Compensated Demand Curve:

- Also known as the **Hicksian demand curve**, it shows how much of a good a consumer would demand if their **real income is adjusted (compensated) to neutralize the income effect**, considering only the substitution effect.
- It isolates the pure effect of price change, eliminating the income effect.

### Key Differences:

| **Feature** | **Ordinary Demand Curve** | **Compensated Demand Curve** |
|---|---|---|
| Effect Considered | Both income and substitution effects | Only substitution effect |
| Derived from | Utility maximization | Expenditure minimization |
| Price Change Response | Consumers react to both effects | Consumers react only to substitution effect |
| Slope | Always downward sloping | Can be steeper or flatter depending on substitution effect |

---

## 1.2 Derivation of Ordinary and Compensated Demand Functions

### Derivation of Ordinary Demand Function:

The **Marshallian demand function** expresses demand as a function of price and income:

$$X = f(P_x, P_y, M)$$

where:

- $X$ = Quantity demanded
- $P_x$ = Price of the good
- $P_y$ = Price of other goods
- $M$ = Consumer's income

To derive it:

1. **Utility Maximization:** Consumers aim to maximize utility **U(X, Y)** subject to their budget constraint:

$$P_x X + P_y Y = M$$

2. **Solving the Lagrangian Function:**

$$L = U(X, Y) + \lambda(M - P_x X - P_y Y)$$

Differentiating and solving for $X$ gives the **Marshallian demand function**.

### Derivation of Compensated Demand Function:

The **Hicksian demand function** expresses demand as a function of price and utility:

$$X = h(P_x, P_y, U)$$

where:

- $X$ = Quantity demanded
- $P_x$ = Price of the good
- $P_y$ = Price of other goods
- $U$ = Fixed level of utility

To derive it:

1. **Expenditure Minimization:** Consumers minimize expenditure **E(P_x, P_y, U)** while maintaining a fixed level of utility.
2. **Setting up the Lagrangian Function:**

$$L = P_x X + P_y Y + \lambda(U - U(X, Y))$$

3. **Solving for $X$** gives the **Hicksian demand function**, which isolates the substitution effect.

**Key Relationship:**
The **Slutsky Equation** links both demand functions:

$$\frac{dX}{dP_x} = \frac{dX^c}{dP_x} - \frac{X}{M}\frac{dM}{dP_x}$$

where $X^c$ represents the compensated demand.

---

### 1.3 Indirect Utility Function

- Represents the **maximum utility** a consumer can achieve at given prices and income.
- Denoted as:

$$V(P_x, P_y, M) = U(X^*, Y^*)$$

where $X^*, Y^*$ are optimal consumption bundles derived from the Marshallian demand function.
- Helps derive **expenditure functions and demand elasticities**.

### Properties of Indirect Utility Function:

1. **Non-Increasing in Prices**: If the price of a good increases, utility decreases.
2. **Increasing in Income**: As income increases, utility increases.
3. **Homogeneous of Degree Zero**: If prices and income increase proportionally, real utility remains unchanged.

---

### 1.4 Income and Substitution Effects of Normal and Inferior Goods

**Income Effect:**

- When the price of a good falls, real income increases, allowing the consumer to buy more.
- For **normal goods**, a decrease in price leads to an increase in demand.
- For **inferior goods**, a decrease in price may reduce demand if the consumer shifts to a superior substitute.

**Substitution Effect:**

- When the price of a good falls, it becomes relatively cheaper, leading consumers to substitute it for other goods.
- Always **negative** (i.e., when price decreases, demand increases).

**Cases:**

- **Normal Goods:** Both **income and substitution effects** work in the same direction, increasing demand.
- **Inferior Goods:** The **income effect** works against the **substitution effect**. If the income effect is strong enough, it may lead to a **Giffen good** scenario, where demand decreases as price falls.

---

### 1.5 Application of Indifference Curve

**1.5.1 Labour-Leisure Trade-Off:**

- A consumer allocates time between **work (labour)** and **leisure** to maximize satisfaction.
- **Income effect**: Higher wages increase income, leading to more leisure (less work).
- **Substitution effect**: Higher wages make leisure more expensive, leading to more work.

**Result:**

- If the **substitution effect** dominates → more work.
- If the **income effect** dominates → more leisure.

**Diagram:**
(Include an indifference curve showing time allocation between work and leisure.)

---

**1.5.2 Cash Subsidy vs Food Transfer:**

- **Cash subsidy**: Increases consumer's purchasing power, allowing choice across all goods.
- **Food transfer**: Directly increases the quantity of food but limits flexibility.
- **Economic Efficiency**: Cash subsidies are preferred as they allow consumers to **maximize utility**.

**Diagram:**
(Include an indifference curve comparing utility under a cash subsidy vs. in-kind food transfer.)

---

## 1.6 Revealed Preference Theory (RPT)

- Developed by **Paul Samuelson**, RPT determines consumer preferences without assuming a utility function.
- Based on actual choices in the market:
  - If a consumer chooses **X over Y**, then X is **revealed preferred**.
  - If X is preferred despite Y being cheaper, X is a **strongly revealed preference**.

**Axioms of RPT:**

1. **Weak Axiom of Revealed Preference (WARP)**:
   - If **X is chosen over Y**, then Y should never be chosen over X at the same price.
2. **Strong Axiom of Revealed Preference (SARP)**:
   - Extends WARP to multiple goods.

**Importance:**

- Does not require utility measurement.
- Forms the basis for demand analysis in empirical economics.

**Demand Curves: ODC and CDC:**

A consumer's ordinary demand curve for a good, also called a Marshallian demand curve, gives the quantity of the good he will buy as a function of its price.

The shape of the ordinary demand curve for a good depends upon the properties of the consumer's utility function, viz., diminishing marginal utility (dMU) in the case of Marshallian utility theory and the diminishing marginal rate of substitution (dMRS) in the case of the indifference curve theory.

For a non-giffen good, the ordinary demand curve would be negatively sloped. The ordinary demand curve may be negatively sloped even in the case of an inferior good if the magnitude of the substitution effect of a change in the price of the good is greater than that of the income effect. A negatively sloped ordinary demand curve for any good, X, has been shown in Fig. 6.35.

**Compensated Demand Curves:**

Imagine a situation in which a consumer is taxed in the case of a fall in the price of a good and subsidized in the case of a rise in its price in such a way as to leave his real income constant at the initial (pre-price change) level.

The demand curve that gives the quantity demanded of the good as a function of its price under these (compensated) conditions is called a compensated demand curve (CDC).

It may be noted here that both the imposition of a tax in the case of a price fall and the grant of a subsidy in the case of a price rise in order to keep the real income of the consumer at the initial level, are known as the compensating variation in income (CVI).

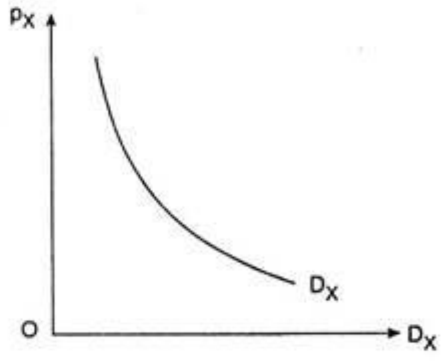



**Fig. 6.35** A consumer's ordinary demand curve for any good X

Compensated demand curves (CDCs) are of two types. First, the CDCs that are obtained on the basis of the method of compensation put forward by Slutsky. These demand curves are called the Slutsky demand curves.

According to Slutsky, after a price change, the consumer should be compensated (taxed or subsidized) in such a way that he might be able to buy also the pre-change equilibrium combination of the goods.

Second, the CDCs that are derived from the method of compensation put forward by Hicks. These curves are called Hicks demand curves. According to Hicks, the consumer should be compensated after a price change in such a way as to make him stay on the 'original' indifference curve on which his pre-change equilibrium point was located.

It follows from the definition of the CDCs that the Slutsky CDC is based on the Slutsky substitution effect, and the Hicks CDC is based on the Hicks substitution effect, of a price change (since the income effect in both the cases has been cancelled by the process of compensation).

Both these CDCs are negatively sloped because of convexity-to-the origin of the consumer's ICs, i.e., because of the diminishing $MRS_X$, y.

When the consumer gets compensated for the price change, the resulting change in the quantity demanded of the good would be of a lesser magnitude. That is why, at any particular price, the CDC (of any variety, Slutsky or Hicks) would be less elastic or more steep than an ordinary (Marshallian) demand curve.

Again, of the Slutsky and Hicks CDCs, the latter would be less elastic than the former because Hicks' method involves compensation of a larger magnitude, or, the Slutsky substitution effect is under-compensatory.

**Derivation of the Marshallian, Slutsky and Hicks Demand Curves from the Consumer's Indifference Curves:**

Now let's explain the derivation of the Marshallian, Slutsky and Hicks demand curves for good X from the consumer's ICs between the goods X and Y with the help of Fig. 6.36.

In Fig. 6.36(a), initially the budget line of the consumer is $L_1M_1$ and his equilibrium point is $E_1$ on $IC_1$. The budget line $L_1M_1$ is obtained for a price of good X, equal to, say, $OP'_x$ (or, $p_x$). So at the point $E_1$, the consumer purchases OA, of X at the price = $OP'_x$.

This price-quantity combination is represented by the point R in Fig. 6.36(b). Now suppose, $p_x$ falls, ceteris paribus, from $OP'_x$ to $OP_x$ (or, $p_x$). If there is no compensation, the consumer's budget line will now rotate from $L_1M_1$ to say, $L_1M_2$. He will now buy $OA_2$ of X at the point $E_2$ on $IC_2$.

So the price-quantity combination now would be ($p_x$, $OA_2$). This is represented by the point $T_1$. Now join the points R and T, by a curve, the Marshallian demand curve is obtained, D. in Fig. 6.36(b).

It may be noted here that the dotted curve passing through the points $L_1$,$E_1$ and $E_2$ in Fig. 6.36(a) is the price-consumption curve (PCC) for good X, and there is a one-to-one correspondence between the points on the PCC and the Marshallian demand curve.

For example, at the point E, on PCC, $p_x$ is implicitly given to be $p'_x$ and demand for X is $OA_1$, and this price- quantity combination is represented by point R on the Marshallian demand curve RD in Fig. 6.36(b).

Similarly, at the point $E_2$ on PCC, $p_x$ is $p_x$ and demand for X is $OA_2$, and this price- quantity combination is represented by the point T, on the Marshallian demand curve. Now, the curve passing through the points like R and T, gives us the Marshallian demand curve that is derived from the PCC.

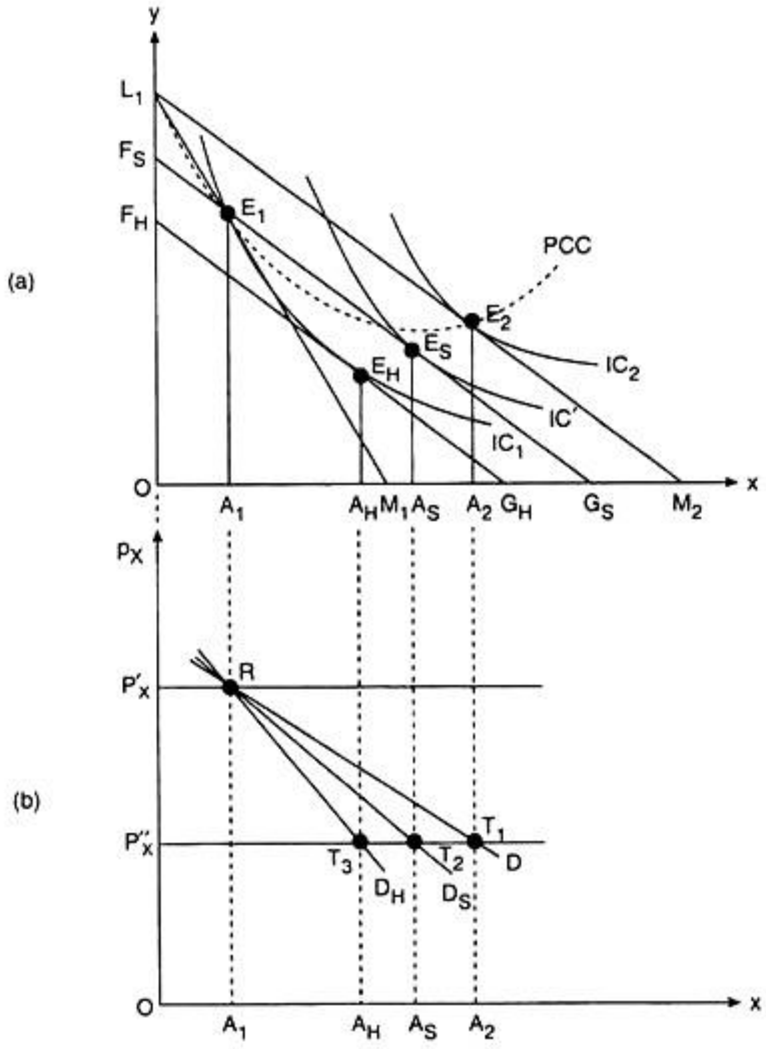



**Fig. 6.36** Derivation of the compensated demand curves from the indifference map of the consumer

The Slutsky and Hicks compensated demand curves is obtained from the indifference curves given in Fig. 6.36(a).

In Fig. 6.36(a), the Slutsky 'compensated' budget line is $F_SG_S$ and the consumer is in equilibrium at the point $E_s$ buying $0A_S$ of good X at $p_x = OP''_X$. This price-quantity combination is the point $T_2$ in Fig.

6.36(b). Therefore, the curve $D_s$, passing through the points R and $T_2$ is the Slutsky (compensated) demand curve.

Lastly, the Hicks 'compensated' budget line is $F_H G_H$ in Fig. 6.36(a). The consumer is in equilibrium at the point $E_H$ on this budget line buying $OA_H$ of good X at $p_X = OP''_X$. This price-quality combination is obtained at the point $T_3$ in Fig. 6.36(b). The curve $D_H$, passing through the points R and $T_3$, is the Hicks (compensated) demand curve.

Is is seen, therefore, how the Marshallian, Slutsky and Hicks demand curves can be derived from the consumer's indifference curves. For the sake of simplicity, draw these curves as straight lines in Fig. 6.36(b).

It is also noted (and illustrated below) that CDCs of both Hicks and Slutsky are less elastic than the ODC. One last point can be mentioned here is that Hicks and Slutsky quantity demanded would not differ substantially, if the price-change is very small. So it may lump $D_S$ and $D_H$ [Fig. 6.36(b)] together and call them simply the compensated demand curves.

**Elasticities of Ordinary and Compensated Demand Curves:**

From the definitions of an individual consumer's ordinary (or Marshallian) demand curve (ODC) and a compensated demand curve (CDC) for some good, say, X, it follows that elasticity of demand (numerical coefficient, e, of price-elasticity of demand) at any price would be smaller on a CDC than on an ODC.

This is because when $p_x$ falls, the consumer would be taxed in both the cases of Hicks and Slutsky compensation, to keep his real income constant.

Therefore, the consumer's purchase either would not have the income effect (as in the Hicks case), or, would have a very small IE (as in the under-compensatory Slutsky case), of the rise in real income due to the price fall. So if X is a normal or superior good, his purchase here would be able to rise at best to a very small extent because of the IE.

His purchase would rise mostly because of the substitution effect (SE) of a fall in the relative price of good X. On the other hand, in the Marshallian case, the consumer's purchase of X would increase owing to both the IE and the SE of the fall in $p_x$.

Therefore, owing to a fall in $p_x$ at any price-quantity combination, the 'compensated' demand would rise by a smaller proportion than the Marshallian demand. That is, at the said price-quantity point, the compensated demand curve would be steeper than the Marshallian demand curve, and e would be smaller in the former case, as has happened at the price of OP'x in Fig. 6.37.

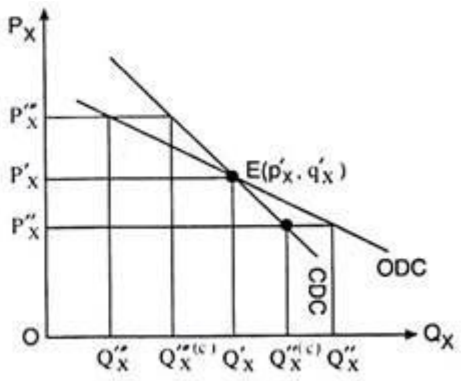



**Fig. 6.37** CDC is less price-elastic than the ODC

In Fig. 6.37, it is shown a CDC and an ODC. As $p_x$ falls from OP’x to OP”x, demand would rise from OQ’x to OQ”x along the ODC and from OQ’$_x$ to OQ”x$^{(c)}$ along the CDC, the magnitude of the latter rise being smaller.



Similarly, as $p_x$ rises from OP’$_x$ to OP”’x , the consumer would be subsidized under compensation, and the effect of a fall in real income of the consumer, i.e., IE would be eliminated or it would be very small (in the Slutsky case).

So under compensation, as $p_x$ rises, the consumer’s demand would fall only because of the SE of a rise in the relative price of X. In the Marshallian case, on the other hand, as $p_x$ rises, demand would fall owing to both IE and SE of the price rise.

That is, as $p_x$ rises, fall in demand for X would be smaller in the ‘compensated’ case than in the Marshallian case, making the CDC steeper again than the ODC and making ‘e’ smaller in the former case and larger in the latter case.

In Fig. 6.37, as $p_x$ rises from OP’x to OP”’x , compensated demand falls by a smaller amount, from OQ’x to OQ”’x$^{(c)}$, than the amount, from OQ’x to OQ”’x, by which the Marshallian demand falls.

From what is explained above, it is obvious that the ODC and CDC would intersect at the point of reference E(p’$_x$, q’$_x$). Then as $p_x$ falls or rises, the compensated demand would rise or fall along a steeper curve and the ordinary or the Marshallian demand would rise or fall along a flatter curve.

## Substitution Effect, Income Effect, Normal and Inferior Goods:

There are two different phenomena underlying a consumer's response to a price drop:

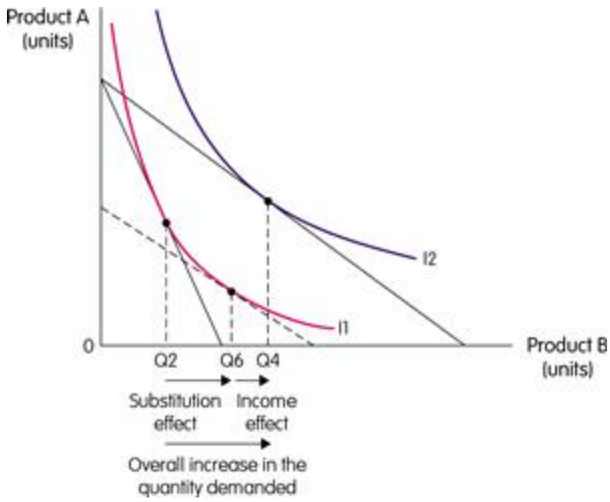
- As the price of a product declines, the lower opportunity cost will induce consumers to buy more of it since it becomes less expensive - even if they have to give up other products. This is called the **substitution effect**.

To isolate this effect diagrammatically, we move the new budget line inwards and parallel until it is tangent to the old indifference curve. The new slope reflects the new relative prices but the utility is the same as it was originally. The substitution effect is Q2Q6. The substitution effect will always lead to more of the relatively cheaper product being demanded.

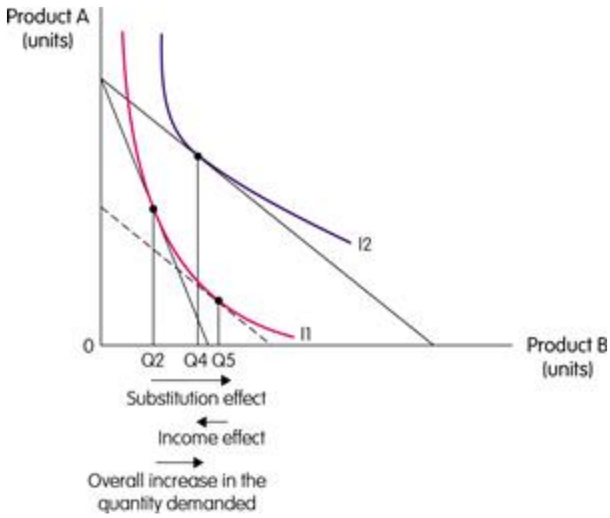
- With a fixed amount of money income, a reduction in the price of a product will increase a consumer's real income - the amount of goods and services consumers are able to purchase. Typically, consumers will respond by purchasing more of the cheaper products (as well as other products). This is called the **income effect**. The income effect is identified by shifting the budget line back outwards again. In this case, this leads to an increase in the quantity demanded of Q6 Q4.

The substitution and income effects will generally work in the same direction, causing consumers to purchase more as the price falls and less as the price rises. The indifference curve can be used to separate these two effects.

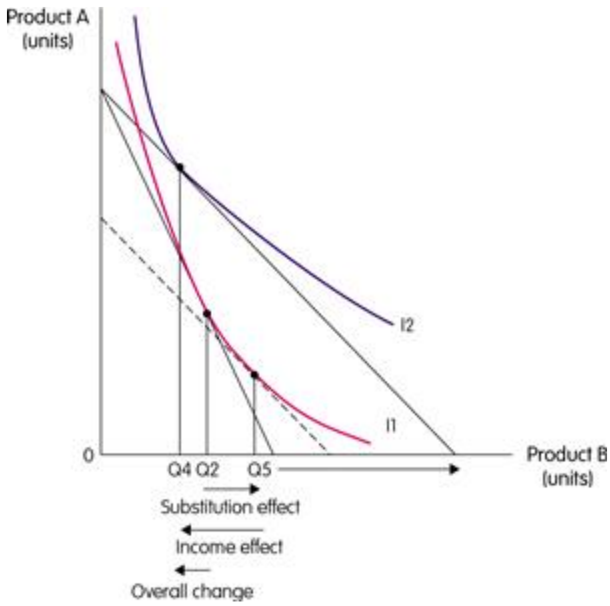
In the case of a **normal good**, higher real income leads to an increase in quantity demanded; this complements the increase due to the substitution effect. This change is shown in the diagram below.

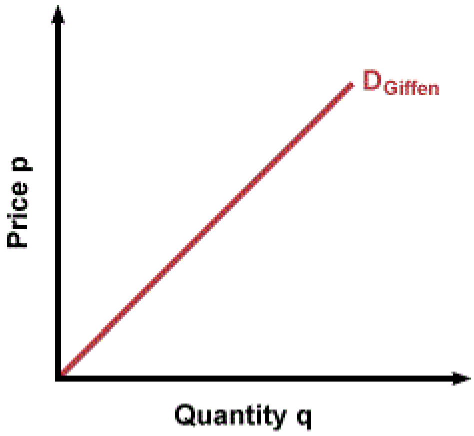



In the case of an **inferior product**, the income effect leads to a fall in the quantity demanded, which will work against the substitution effect. In the following diagram the substitution effect is Q2 Q5; the income effect is Q5 Q4. However, the substitution effect outweighs the income effect and overall the quantity demanded rises. The overall change in quantity demanded results in an increase of Q2 Q4. This means the demand curve is downward-sloping, because a price fall increases the quantity demanded.




When a good is inferior and the income effect outweighs the substitution effect, it is called a **Giffen good**. This is, however, unlikely, because the substitution effect is almost always stronger than the income effect.




| The effect of a fall in price | | | |
|---|---|---|---|
| | **Substitution effect** | **Income effect** | **Overall** |
| Normal good | Increases quantity demanded | Increases quantity demanded | Increases quantity demanded; downward sloping demand curve |
| Inferior good | Increases quantity demanded | Decreases quantity demanded | Increases quantity demanded; downward sloping demand curve |
| Giffen good | Increases quantity demanded | Decreases quantity demanded | Decreases quantity demanded; upward sloping demand curve |

Another exception is the case where an increase in price causes an increase in demand. This results in an upward-sloping demand curve, and the good is called a **Veblen good**.

One possible justification for a Veblen good is that people associate higher prices with status, luxury, and quality, so that a higher price might increase the perceived value of a good.

## Individual's Choice between Income and Leisure (Explained with Diagram):

Indifference curve analysis can be used to explain an individual's choice between income and leisure and to show why higher overtime wage rate must be paid if more hours of work is to be obtained from the workers.

It is important to note that income is earned by devoting some of the leisure time to do some work. That is income is earned by sacrificing some leisure.

The greater the amount of this sacrifice of leisure, that is, the greater the amount of work done, the greater income an individual earns.

Further, income is used to purchase goods, other than leisure for consumption. Leisure time can be used for resting, sleeping, playing, listening to music on radios and television etc. all of which provide satisfaction to the individual. Therefore, in economics leisure is regarded as a normal commodity the enjoyment of which yields satisfaction to the individual.

While leisure yields satisfaction to the individual directly, income represents general purchasing power capable of being used to buy goods and services for satisfaction of various wants. Thus income provides satisfaction indirectly. Therefore, we can draw indifference curves between income and leisure, both of which give satisfaction to the individual.

Indifference maps between income and leisure is depicted in Figure 11.12 and have all the usual properties o/indifference curves. They slope downward to the right, are convex to the origin and do not intersect. Each indifference curve represents various alternative combinations of income and leisure which provide equal level of satisfaction to the individual and the farther away an indifference curve is from the origin, the higher the level of satisfaction it represents for the individual.

The slope of the indifference curve measuring marginal rate of substitution between leisure and income ($MRS_{LM}$) shows the tradeoff between income and leisure. This trade-off means how much income the individual is willing to accept for one hour sacrifice of leisure time.

In geometric terms, it will be seen from Figure 11.12 that on indifference curve $IC_1$ at point A the individual is willing to accept $\Delta M(=AC)$ income for sacrificing an hour ($\Delta L$) or BC of leisure. Thus the trade-off between income and leisure at this point is $\Delta M/\Delta L$. At different income-leisure levels, the trade-off between leisure and income varies.

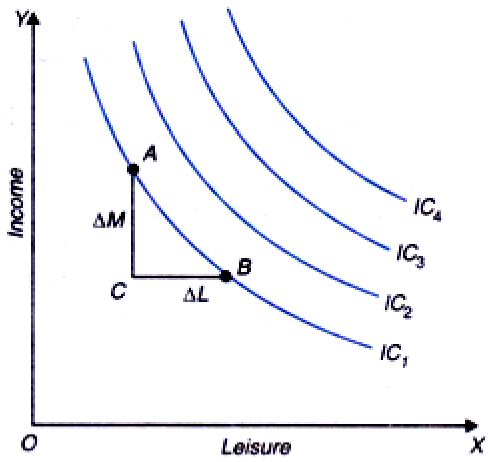



Fig.11.12. *Trade-off Curves: Indifference Curves between Income and Leisure*

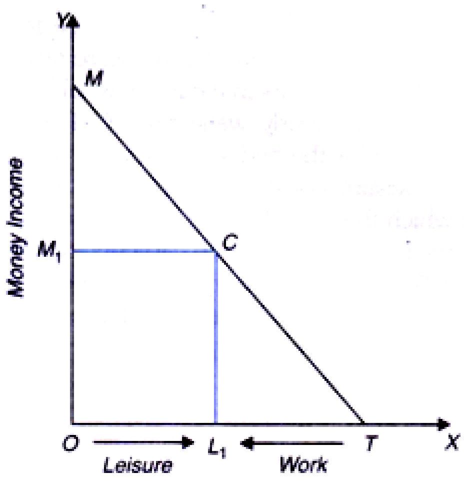



Fig. 11.13. *Income-Leisure Constraint*

Indifference curves between income and leisure are therefore also called trade-off curves.

**Income-Leisure Constraint:**

However, the actual choice of income and leisure by an individual would also depend upon what is the market rate of exchange between the two, that is, the wage rate per hour of work. It is worth noting that wage rate is the opportunity cost of leisure.

In other words, to increase leisure by one hour, an individual has to forego the opportunity of earning income (equal to wage per hour) which he can earn by doing work for an hour. This leads us to income-leisure constraint which together with the indifference map between income and leisure would determine the actual choice by the individual.

The maximum amount of time available per day for the individual is 24 hours. Thus, the maximum amount of leisure time that an individual can enjoy per day equals 24 hours. In order to earn income for satisfying his wants for goods and services, he will devote some of his time to do work.

Consider Figure 11.13 where leisure is measured in the rightward direction along the horizontal axis and the maximum leisure time is OT (equal to 24 hours). If the individual can work for all the 24 hours in a day, he would earn income equal to OM. Income OM equals OT multiplied by the hourly wage rate (OM = OT.w) where w represents the wage rate.

The straight line MT is the budget constraint, which in the present context is generally referred to as income-leisure constraint which shows the various combinations of income and leisure among which the individual will have to make a choice. Thus, if a person chooses combination C, this means that he has $OL_1$ amount of leisure time and $OM_1$ amount of income. He has earned $OM_1$ amount of income by working $TL_1$ hours of work. Choice of other points on income-leisure line MT will show different amounts of leisure, income and work.

Income OM = OT. W

OM/OT = w

Thus, the slope of the income-leisure curve OM/OT equals the wage rate.

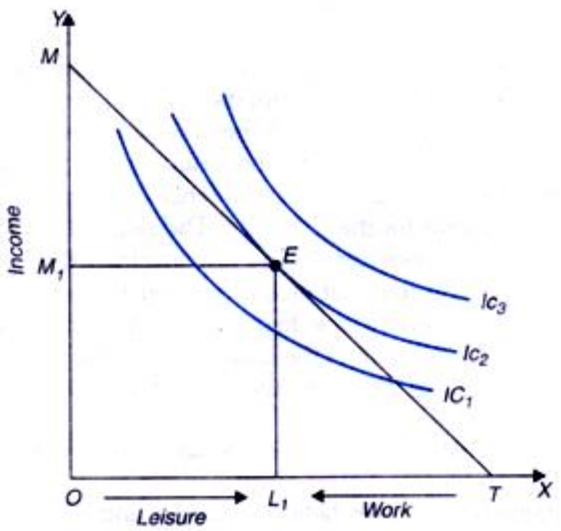



*Fig. 11.14 Income-Leisure Equilibrium*

**Income-Leisure Equilibrium:**

Now, we can bring together the indifference map showing ranking of preferences of the individual between income and leisure and the income-leisure line to show the actual choice of leisure and income by the individual in his equilibrium position. We will further show how much work effort (i.e. supply of labour in terms of hours worked) he would put in this optimal situation. Our analysis is based on two assumptions. First, he is free to work as many hours per day as he likes. Second, wage rate is the same irrespective of the number of hours he chooses to work.

Figure 11.14 displays income-leisure equilibrium of the individual. With the given wage rate, the individual will choose a combination of income and leisure lying on the income-leisure line MT that maximises his satisfaction. It will be seen from Figure 11.14 that the given income- leisure line MT is tangent to the indifference curve $IC_2$ at point E showing choice of $OL_1$ of leisure and $OM_1$ of income.

In this optimal condition, income- leisure trade off (i.e. MRS between income and leisure) equals the wage rate (i.e., that is, the market exchange rate between the two. In this equilibrium position the individual works for $TL_1$ hours per day ($TL_1$ = OT- $OL_1$). Thus, he has worked for $TL_1$, hours to earn $OM_1$ amount of income.

**Income Effect and Substitution Effect of the Change in Wage Rate:**

Now the supply curve of labour does not always slope upward as shown in Fig. 11.16. It can slope or bend backward too which implies that at a higher wage rate, the individual will supply less labour (i.e. will work less hours). Under what conditions supply curve of labour (i.e. work- hours) slopes upward and under what circumstances it bends backward can be explained in termsof income effect and substitution effect of a change in wage rate.

As in case of change in price, rise in wage rate has both the substitution effect and income effect. The net combined effect on the supply of labour (hours worked) depends on the magnitude of the substitution effect and income effect of the rise in wage rate. It is important to note that leisure is a normal commodity which means that increase in income leads to the increase in leisure enjoyed (i.e. less work-hours supplied). That is, income effect of the rise in wage rate on leisure is positive, that is, leads to the increase in the hours of leisure enjoyed (that is, tends to decrease labour supply).

On the other hand, the rise in wage rate increases the opportunity cost or price of leisure, that is, it makes enjoyment of leisure relatively more expensive. Therefore, as a result of rise in wage rate individual substitutes work (and therefore income) for leisure which leads to the increase in supply of labour. This is a substitution effect of the rise in wage rate which tends to reduce leisure and increase labour supply (i.e. number of hours worked). It is thus clear that for an individual supplier of labour, income effect and substitution effect work in opposite directions.

Whereas income effect of the rise in wage rate tends to reduce supply of labour substitution effect tends to increase it. If the income effect is stronger than the substitution effect, the net combined effect of rise in wage rate will be to reduce labour supply. On the other hand, if substitution effect is relatively larger than the income effect, the rise on wage rate will increase labour supply.

How the effect of rise in wage rate is split up into income effect and substitution effect is shown in Fig 11.17. In this figure we measure money income on the Y- axis and leisure (reading from left to right) and labour supply (reading from right to left) on the X-axis. Suppose to begin with the wage rate is $w_0$ and if all the available hours OT are used to do work, $OM_0$ money income is earned. This gives us $TM_0$ as the budget constraint or which in the present context is also called leisure-income constraint. It will be seen from Figure 11.17 that $TM_0$ is tangent to indifference curve $IC_1$ between leisure and income at point R.

Thus, with wage rate $W_0$ the individual is in equilibrium when he enjoys $OL_0$ leisure and therefore he is supplying $TL_0$ work hours of labour. Now suppose that wage rate rises to $w_0$ with the result that income- leisure constraint line rotates to $TM_1$. Now, with $TM_1$ as new income-leisure constraint line, the individual is in equilibrium at point H at which he supplies $TL_1$ work-hours of labour which are less than $TL_0$.

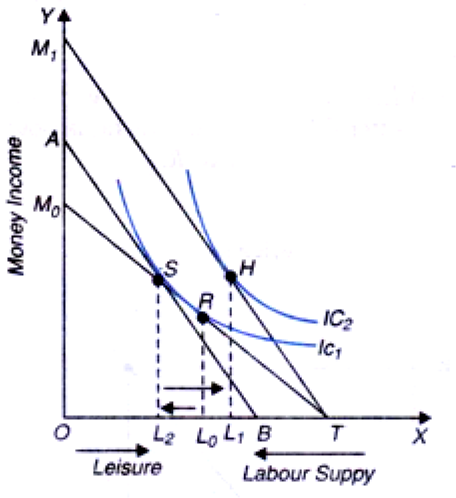



Fig. 11.17 Decomposing the Effect of Rise in Wage Rate into Income Effect and Substitution Effect

Thus, with the rise in wage rate, supply of labour has decreased by $L_0L_1$. To break up this wage effect on labour supply, we reduce his money income by compensating variation in income. To do so we take away so much income from the individual that he comes back to the original indifference curve $IC_1$. AB is such line obtained after reducing his money income by compensating variation. AB is tangent to indifference curve $IC_1$ at point S at which he supplies $TL_2$ hours for work.

This shows with change in wage rate from $w_0$ to $w_1$, resulting in leisure becoming relatively more expensive, he substitutes work (i.e. labour supply) $L_0L_2$ for leisure. This is substitution effect which tends to increase labour supply by $L_0L_2$, Now, if the money taken from him is given back to him so that the income-leisure line again shifts back to $TM_1$. With $TM_1$, he reaches his old equilibrium position at point H where he supplies $TL_1$ work- hours. Thus, movement from point S to H represents the income effect of the rise in wage rate and as a result labour supply decrease by $L_2L_1$.

Thus, while income effect of the increase in wage rate causes decrease in labour supply by $L_2L_1$ the substitution effect causes increase in labour supply by $L_2L_1$. It will be seen from Fig. 11.17 that in this case income effect is stronger than substitution effect so that the net result is reduction in labour supply by $L_0L_1$ work-hours and therefore in this case labour supply curve bends backward. Now, if substitution effect had been larger than income effect, work-hours supplied would have increased as a result of rise in wage rate and labour supply curve would slope upward.

**Backward Bending Supply Curve of Labour:**

It may, however, be noted that on theoretical grounds it cannot be predicted which effect will be stronger. It has, however, been empirically observed that when the wage rate is small so that the demand for more income or goods and services is very strong, substitution effect is larger than the income effect so that the net effect of rise in wage rate will be to reduce leisure and increase the supply of labour.

But when he is already supplying a large amount of labour and is earning sufficient income, further increases in wage rate may induce the individual to demand more leisure so that income effect may outweigh the substitution effect at higher wage rates.

This implies that at higher wage rates, labour supply may be reduced in response to further rise in wage rates. This means up to a point substitution effect is stronger than income effect so that labour supply curve slopes upward, but beyond that at higher wage rates, supply curve of labour bends backward.

This is illustrated in Fig 11.18 where in panel (a) wage offer curve is shown, and in panel (b) supply curve of is drawn corresponding to leisure-work equilibrium in panel (a). Thus, to start with at wage rate $w_0$ (i.e. $TM_0$ as budget constraint) $L_0$ amount of work-hours (labour) are supplied. This is directly plotted against the wage rate $w_0$ in panel (b) of Fig. 11.18. When the wage rate rise to budget constraint becomes $TM_1$ in panel (a) of Fig. 11.18 the greater amount of labour $L_1$ is supplied.

Amount of labour $L_1$ is directly plotted against higher wage rate $w_1$ in panel (b) of Fig. 11.18. With the further increase in wage rate to $w_2$, the income-leisure constraint rotates to $TM_2$ and the individual is in equilibrium when he supplies $L_1$ work-hours which are smaller than $L_1$. Thus, with the rise in wage rate above $w_1$, labour supply decreases. In other words, up to wage rate $w_1$, labour supply curve slopes upward and beyond that it starts bending backward. This is quite evident from panel (b) of Fig. 11.18.

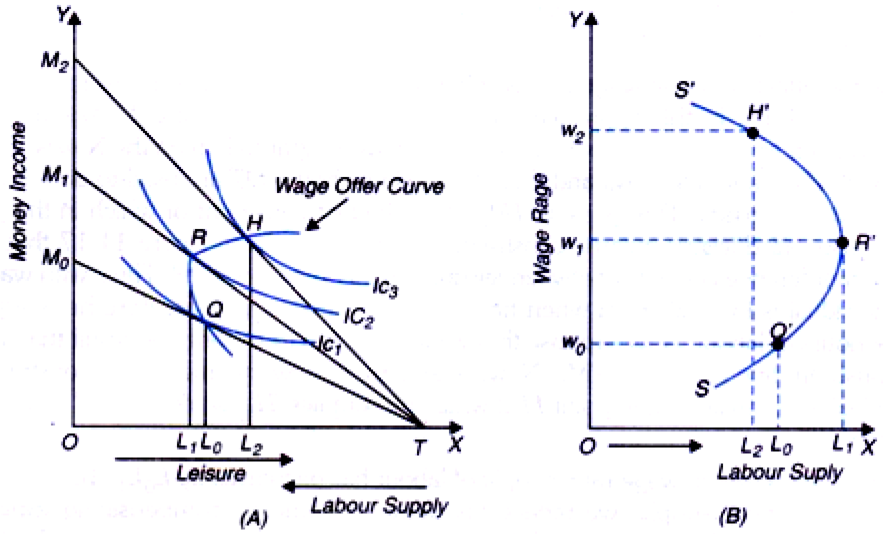



Fig. 11.18. *Backward-Bending Supply Curve of Labour*

ONE MARK:

**1. What is an ordinary demand curve?**

✅ An **ordinary demand curve** (Marshallian demand curve) shows the relationship between price and quantity demanded, considering both **income and substitution effects**.

**2. What is a compensated demand curve?**

✅ A **compensated demand curve** (Hicksian demand curve) shows the quantity demanded when real income is adjusted to keep utility constant, isolating only the **substitution effect**.

**3. What is the difference between ordinary and compensated demand?**

✅ **Ordinary demand** considers **both income and substitution effects**, while **compensated demand** considers only the **substitution effect**.

**4. What is the Marshallian demand function?**

✅ The **Marshallian demand function** expresses demand as a function of price and income:

X=f(Px,Py,M)

**5. What is the Hicksian demand function?**

✅ The **Hicksian demand function** expresses demand as a function of price and utility:

X=h(Px,Py,U)

**6. What is an indirect utility function?**

✅ It represents the **maximum utility** a consumer can achieve given prices and income.

**7. What is the Slutsky equation?**

✅ The **Slutsky equation** separates total price effect into **income and substitution effects**:

dX/dPx = dXc/dPxd = XdM/MdPx

**8. What happens when the price of a normal good falls?**

✅ **Both income and substitution effects increase demand**.

**9. What happens when the price of an inferior good falls?**

✅ **The substitution effect increases demand, but the income effect decreases it**.

**10. What is a Giffen good?**

✅ A **Giffen good** is an **inferior good** where the **negative income effect outweighs the substitution effect**, leading to a **rise in price increasing demand**.

**11. What is the income effect?**

✅ It shows how demand changes when **real income changes**, holding prices constant.

**12. What is the substitution effect?**

✅ It shows how demand changes when a good's price changes **relative to other goods**, holding utility constant.

**13. What is the labour-leisure trade-off?**

✅ It refers to how individuals allocate their time between **work (labour) and leisure** based on wages.

**14. How does a rise in wages affect the labour-leisure trade-off?**

✅ **Substitution effect**: More work, less leisure.
✅ **Income effect**: More leisure, less work.

**15. What is the effect of cash subsidies on consumer choice?**

✅ **Cash subsidies** increase consumer choice, allowing them to maximize **utility**.

**16. What is the effect of food transfer on consumer choice?**

✅ **Food transfer** restricts choice as consumers receive a fixed amount of food instead of cash.

**17. Which is better: cash subsidy or food transfer?**

✅ **Cash subsidy** is better because it gives **more flexibility** and higher utility.

**18. What is the Revealed Preference Theory (RPT)?**

✅ Developed by **Paul Samuelson**, it determines preferences based on actual market choices instead of utility functions.

**19. What is the Weak Axiom of Revealed Preference (WARP)?**

✅ If a consumer prefers **X over Y**, they should never choose **Y over X** at the same price.

**20. What is the Strong Axiom of Revealed Preference (SARP)?**

✅ It extends **WARP** to multiple goods, ensuring consistency in consumer choices.

**21. What is an expenditure function?**

✅ It shows the **minimum income** required to achieve a certain level of **utility** at given prices.

**22. What is the key assumption of the Revealed Preference Theory?**

✅ Consumers make **rational** and **consistent** choices based on preferences.

**23. What is the Engel Curve?**

✅ It shows the relationship between **income and quantity demanded** of a good.

**24. What is the price elasticity of demand?**

✅ It measures **how much quantity demanded changes** due to a change in **price**.

**25. What happens to demand when both price and income double?**

✅ Since utility remains unchanged, demand remains the **same** due to the **homogeneity property**.

TWO MARK:

**1. What is the difference between the ordinary and compensated demand curve?**

The **ordinary demand curve** (Marshallian demand curve) represents the quantity demanded at different prices while considering both **income and substitution effects**. On the other hand, the **compensated demand curve** (Hicksian demand curve) isolates the **substitution effect** by adjusting income to maintain the same level of utility.

**2. How is the ordinary demand function derived?**

The **ordinary demand function** is derived by maximizing the **utility function** subject to a given **budget constraint**. It expresses demand as a function of **income and prices**:

$$X = f(P_x, P_y, M)$$

where $P_x$ and $P_y$ are prices of goods, and $M$ is income.

**3. How is the compensated demand function derived?**

The **compensated demand function** is derived from the **expenditure minimization problem**, where the consumer chooses a bundle that provides a fixed level of utility at the lowest cost. It is expressed as:

$$X = h(P_x, P_y, U)$$

where **utility (U)** is constant.

**4. What is the indirect utility function?**

The **indirect utility function** represents the **maximum level of utility** a consumer can achieve given their **income and market prices**. It is derived from the ordinary demand function and shows how utility changes with **income and prices**.

**5. Explain the income effect for normal and inferior goods.**

The **income effect** shows how demand changes when real income changes.

- For a **normal good**, an increase in income **increases demand**.
- For an **inferior good**, an increase in income **decreases demand**, as consumers shift to superior alternatives.

**6. Explain the substitution effect for normal and inferior goods.**

The **substitution effect** occurs when a price change alters the relative attractiveness of goods.

- For both **normal and inferior goods**, a decrease in price **increases demand** due to substitution towards the cheaper good.
- The substitution effect is **always negative** (inverse relationship with price).

**7. How does the Slutsky equation explain consumer behavior?**

The **Slutsky equation** breaks the total price effect into **income effect and substitution effect**:

$$\frac{dX}{dP_x} = \frac{dX^c}{dP_x} - \frac{X}{M}\frac{dM}{dP_x}$$

where $X$ is demand, $P_x$ is price, and $M$ is income.

**8. What is a Giffen good?**

A **Giffen good** is an **inferior good** where the **negative income effect dominates the substitution effect**, causing demand to increase as price rises. An example is **staple food items like rice or bread** among poor consumers.

**9. How does the labour-leisure trade-off work?**

The **labour-leisure trade-off** analyzes how individuals allocate time between work and leisure. If wages rise:

- **Substitution effect**: More work, less leisure.
- **Income effect**: More leisure, less work.

**10. How does an increase in wages affect the labour-leisure trade-off?**

When wages rise, the **substitution effect** encourages more work (higher opportunity cost of leisure), while the **income effect** makes individuals want more leisure (higher income reduces work necessity). The final outcome depends on which effect dominates.

**11. What is the difference between a cash subsidy and a food transfer?**

A **cash subsidy** gives consumers money, increasing their purchasing power and allowing them to choose any good. A **food transfer** provides specific goods (e.g., free rice), restricting consumer choice. Economists argue that **cash subsidies lead to higher overall welfare**.

**12. Why is a cash subsidy considered better than a food transfer?**

A cash subsidy allows consumers to spend on their **most preferred combination of goods**, maximizing their **utility**. A food transfer limits choice and might result in **excess supply** of a good the consumer does not need.

**13. What is the Revealed Preference Theory?**

Developed by **Paul Samuelson**, the **Revealed Preference Theory (RPT)** suggests that consumer preferences can be understood by analyzing actual market choices, rather than using **hypothetical utility functions**.

**14. What is the Weak Axiom of Revealed Preference (WARP)?**

The **Weak Axiom of Revealed Preference** states that if a consumer chooses **good X over good Y** at a given price, they should **never** choose **Y over X** when the price remains unchanged.

**15. What is the Strong Axiom of Revealed Preference (SARP)?**

The **Strong Axiom of Revealed Preference** extends WARP to multiple goods, ensuring that consumer choices remain **consistent** across different price and income levels.

**16. What is an expenditure function?**

An **expenditure function** shows the **minimum amount of money** required to achieve a given level of **utility** at given prices.

**17. What is an Engel Curve?**

The **Engel Curve** illustrates the relationship between **income and quantity demanded**. For **normal goods**, demand increases with income, while for **inferior goods**, demand decreases with income.

**18. How do consumers maximize utility?**

Consumers maximize utility by choosing a combination of goods such that the **marginal rate of substitution (MRS)** equals the **price ratio**:

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

**19. What is price elasticity of demand?**

Price elasticity of demand measures how **quantity demanded changes** in response to a change in **price**:

$$E_d = \frac{\%\Delta Q_d}{\%\Delta P}$$

**20. What is cross-price elasticity of demand?**

Cross-price elasticity measures how the **demand for one good** responds to a change in the **price of another good**.

**21. What is income elasticity of demand?**

Income elasticity of demand measures how **quantity demanded** changes when **income changes**:

$$E_y = \frac{\%\Delta Q}{\%\Delta Y}$$

**22. What happens when both price and income double?**

When both price and income **double**, demand remains **unchanged** due to the **homogeneity property** in consumer theory.

**23. What is a luxury good?**

A **luxury good** has an **income elasticity greater than 1**, meaning demand increases **more than proportionally** as income rises.

**24. What is the price consumption curve (PCC)?**

The **Price Consumption Curve (PCC)** shows the set of **optimal consumption bundles** as the **price of a good changes**, holding income constant.

**25. What is the income consumption curve (ICC)?**

The **Income Consumption Curve (ICC)** shows the set of **optimal consumption bundles** as **income changes**, holding prices constant.

**26. What is the substitution effect of a price change?**

The **substitution effect** occurs when a price change makes one good **relatively cheaper**, leading to increased demand for that good, even if real income remains constant.

**27. What is the budget constraint?**

A **budget constraint** represents the possible combinations of goods a consumer can buy given their **income and prices**:

$$P_x X + P_y Y = M$$

**28. What happens to the budget line when income increases?**

When income **increases**, the **budget line shifts outward**, allowing the consumer to buy more of both goods.

**29. How does an indirect tax affect demand?**

An **indirect tax** (e.g., GST) **raises the price of goods**, leading to a fall in demand due to a **negative substitution and income effect**.

**30. What is the consumer surplus?**

**Consumer surplus** is the difference between the **maximum price a consumer is willing to pay** and the **actual price paid**. It measures **consumer welfare**.

BOARD ANSWER:

**Q1. Differentiate between the Ordinary and Compensated Demand Curve.**

| Basis | Ordinary Demand Curve (Marshallian) | Compensated Demand Curve (Hicksian) |
|---|---|---|
| **Definition** | Shows demand considering both income and substitution effects. | Shows demand isolating only the substitution effect. |
| **Income Effect** | Included | Excluded (compensates income to keep utility constant). |
| **Derivation** | Derived from the utility maximization approach. | Derived from the expenditure minimization approach. |
| **Curve Shape** | Always downward sloping. | Can be steeper or flatter than the ordinary demand curve. |
| **Application** | Used in practical demand analysis. | Used in theoretical and welfare economics. |

**Diagram:**

**Ordinary vs. Compensated Demand Curve follow the theory part, in the above.**

---

**Q2. What is the Indirect Utility Function? Explain.**

**Definition:**
The **Indirect Utility Function** expresses the **maximum utility** a consumer can achieve given **income (M)** and **market prices (P_x, P_y)**. It represents utility as a function of **prices and income** rather than quantities.

**Formula:**

$$V(P_x, P_y, M) = U(X^*, Y^*)$$

where $X^*, Y^*$ are the optimal quantities obtained from the demand function.

**Explanation:**

- It helps in **welfare economics** by showing how utility changes when **income and prices change**.
- It is derived from the **utility maximization** problem and helps in **expenditure function derivation**.

---

**Q3. Explain the Income and Substitution Effect for Normal and Inferior Goods.**

**Income Effect:**

- **Normal Goods:** As income increases, demand increases (positive effect).
- **Inferior Goods:** As income increases, demand decreases (negative effect).

**Substitution Effect:**

- Always negative (as price falls, demand increases).

- Both normal and inferior goods follow the same substitution effect.

**Diagram:**

**Income & Substitution Effect on a Normal Good - follow the theory part, in the above.**

**Conclusion:**

- For **normal goods**, income and substitution effects work in the **same direction**.
- For **inferior goods**, the income effect **partially offsets** the substitution effect.

---

**Q4. Explain the Labour-Leisure Trade-Off with a Diagram.**

**Concept:**
The **labour-leisure trade-off** explains how individuals choose between **working hours (labour)** and **free time (leisure)** based on **wage rate changes**.

**Effects of Wage Increase:**

- **Substitution Effect:** Higher wages make work more attractive, reducing leisure.
- **Income Effect:** Higher income makes leisure more affordable, reducing work hours.

**Diagram:**

**Labour-Leisure Trade-Off - follow the theory part, in the above.**

☑ *Graph showing budget constraint shift, work hours on X-axis, and wage rate on Y-axis.*

**Conclusion:**

- If **substitution effect dominates**, people work more.
- If **income effect dominates**, people work less and enjoy more leisure.

---

**Q5. What is the Revealed Preference Theory? Explain WARP and SARP.**

**Definition:**
Developed by **Paul Samuelson**, the **Revealed Preference Theory (RPT)** states that a consumer's preference can be understood by their **actual choices**, rather than hypothetical utility functions.

**Axioms:**

1. **Weak Axiom of Revealed Preference (WARP):**
   - If a consumer chooses **A over B**, they should not later choose **B over A** when A is still affordable.
2. **Strong Axiom of Revealed Preference (SARP):**
   - Extends WARP by ensuring **consistent consumer choices** across different budget constraints.

**Application:**

- Used in **market demand analysis** to check **consumer rationality**.

- Supports **empirical economic studies** without requiring utility functions.

---

**Q6. Differentiate between Cash Subsidy and Food Transfer.**

| Basis | Cash Subsidy | Food Transfer |
|---|---|---|
| **Definition** | Consumers receive money to spend freely. | Consumers receive food items directly. |
| **Consumer Choice** | High flexibility (can buy anything). | Limited to allocated goods. |
| **Utility Maximization** | Higher (consumers can choose optimally). | Lower (forced consumption). |
| **Market Impact** | Stimulates demand in multiple sectors. | Affects only the food sector. |

**Conclusion:**
Cash subsidies **maximize consumer satisfaction**, while food transfers **restrict choice** but ensure food security for targeted populations.

**Q7. Derive the Ordinary Demand Function.**

**Definition:**
The **Ordinary Demand Function**, also called the **Marshallian Demand Function**, expresses the quantity of a good demanded as a function of its price, the price of other goods, and consumer income.

**Derivation Steps:**

1. **Utility Maximization Condition:**
   - Consumers aim to maximize their utility $U(X, Y)$ subject to a budget constraint:

$$P_x X + P_y Y = M$$

2. **Lagrangian Method:**
   - The Lagrange function is:

$$L = U(X, Y) + \lambda(M - P_x X - P_y Y)$$

3. **First Order Conditions (FOCs):**
   - Differentiating and solving for $X^*$ and $Y^*$ gives the demand functions:

$$X^* = f(P_x, P_y, M)$$

$$Y^* = g(P_x, P_y, M)$$

UNIT – 2

The theory of **cost and production functions** is fundamental in economics, helping us understand how firms transform inputs into outputs and how costs vary with production levels. This unit covers key concepts such as **homogeneous and homothetic production functions, Cobb-Douglas and CES production functions, elasticity of factor substitution, expansion path, and the derivation of cost functions from production functions.**

---

## 1. Homogeneous and Homothetic Production Function

### Homogeneous Production Function

A **homogeneous production function** exhibits a proportional change in output when all inputs are scaled by the same proportion. Mathematically, a production function f(K,L)f(K, L)f(K,L) is homogeneous of degree **'r'** if:

$$f(tK, tL) = t^r f(K, L)$$

- If $r = 1$, the function shows **constant returns to scale (CRS)**.
- If $r > 1$, it shows **increasing returns to scale (IRS)**.
- If $r < 1$, it shows **decreasing returns to scale (DRS)**.

### Homothetic Production Function

A **homothetic production function** is a more general form where the ratio of inputs used depends only on the ratio of marginal products. A function is **homothetic** if it can be written as a **monotonic transformation** of a homogeneous function.

$$F(K, L) = g[f(K, L)]$$

Key properties:

- **All homogeneous functions are homothetic**, but not vice versa.
- The **isoquants** of homothetic functions maintain the same shape but may shift due to scale changes.

---

## 2. Cobb-Douglas and CES Production Function

### Cobb-Douglas Production Function

The **Cobb-Douglas (C-D) production function** is widely used in economics to describe how output (Q) depends on inputs like capital (K) and labor (L):

$$Q = AK^{\alpha}L^{\beta}$$

Where:

- $A$ = Technology parameter
- $\alpha$ = Output elasticity of capital
- $\beta$ = Output elasticity of labor
- $\alpha + \beta$ determines **returns to scale**

If $\alpha + \beta = 1$, CRS; if $\alpha + \beta > 1$, IRS; if $\alpha + \beta < 1$, DRS.

**Properties of the Cobb-Douglas Function**

- It is **homogeneous of degree** (α+β)
- Exhibits **diminishing marginal returns** to individual factors.
- The elasticity of substitution is **constant and equal to 1**.

**CES (Constant Elasticity of Substitution) Production Function**

The **CES production function** generalizes C-D by allowing for different degrees of **substitutability between capital and labor**

$$Q = A\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-\frac{1}{\rho}}$$

Where:

- $\delta$ = Distribution parameter between capital and labor
- $\rho$ = Substitution parameter
- Elasticity of substitution $\sigma = \frac{1}{1+\rho}$

**Properties of CES Function**

- If $\rho = 0$, it **reduces to Cobb-Douglas**.
- If $\rho \to \infty$, capital and labor become **perfect substitutes**.
- If $\rho \to -1$, they are **perfect complements**.

---

**3. Elasticity of Factor Substitution**

The **elasticity of substitution (σ)** measures how easily one input (capital or labor) can be substituted for another while maintaining the same output.

$$\sigma = \frac{d(K/L)}{d(MRTS)}$$

Where **MRTS (Marginal Rate of Technical Substitution)** is:

$$MRTS = \frac{MP_K}{MP_L}$$

- If $\sigma = 1$, Cobb-Douglas function applies.
- If $\sigma > 1$, inputs are easily substitutable.
- If $\sigma < 1$, substitution is difficult.

### 4. Expansion Path

The **expansion path** shows the optimal combination of inputs as a firm's budget expands. It is derived from **cost minimization** and follows the equation:

$$\frac{MP_K}{MP_L} = \frac{P_K}{P_L}$$

Where:

- $MP_K$ and $MP_L$ are marginal products.
- $P_K$ and $P_L$ are prices of capital and labor.

### Types of Expansion Paths

1. **Linear Expansion Path**: For **homothetic** production functions, expansion paths are **straight lines**.
2. **Curved Expansion Path**: When input proportions change with scale, the path is **nonlinear**.

---

### 5. Derivation of Cost Function from Production Function

A **cost function** represents the minimum cost of producing a given output. The steps for derivation are:

#### Step 1: Define the Production Function

For Cobb-Douglas:

$$Q = AK^{\alpha}L^{\beta}$$

#### Step 2: Express the Cost Function

Total Cost (TC) is the sum of input costs:

$$TC = P_K K + P_L L$$

Where $P_K$ and $P_L$ are input prices.

#### Step 3: Solve for Cost-Minimizing Inputs

From cost minimization:

$$MRTS = \frac{MP_K}{MP_L} = \frac{P_K}{P_L}$$

Using this, we derive the **conditional demand functions for K and L**, then substitute into the cost function.

#### Step 4: Get the Cost Function

For Cobb-Douglas:

$$C(Q) = AQ^{\frac{1}{(\alpha+\beta)}} P_K^{\frac{\beta}{(\alpha+\beta)}} P_L^{\frac{\alpha}{(\alpha+\beta)}}$$

ONE MARK:

1. **What is a production function?**
→ A production function represents the relationship between inputs (capital and labor) and output in a firm.

2. **What does a homogeneous production function imply?**
→ A homogeneous production function implies that if all inputs are scaled by the same proportion, output changes by a fixed multiple of that proportion.

3. **What is the degree of homogeneity for a constant return to scale (CRS) production function?**
→ The degree of homogeneity is **one (1)**.

4. **What is a homothetic production function?**
→ A homothetic production function is one where the marginal rate of technical substitution (MRTS) depends only on the ratio of inputs, not on their absolute levels.

5. **What is the general form of the Cobb-Douglas production function?**
→ $Q = AK^{\alpha}L^{\beta}$, where $K$ = capital, $L$ = labor, $A$ = technology, and $\alpha, \beta$ = output elasticities.

6. **What does the sum $(\alpha + \beta)$ indicate in the Cobb-Douglas function?**
→ It indicates the **returns to scale** of production.

7. **If $\alpha + \beta > 1$ in the Cobb-Douglas function, what does it imply?**
→ It implies **increasing returns to scale (IRS)**.

5. **What does the Constant Elasticity of Substitution (CES) production function allow?**
→ It allows different degrees of **substitutability between capital and labor**.

6. **What is the elasticity of substitution in the Cobb-Douglas function?**
→ The elasticity of substitution is **equal to 1**.

7. **What does the elasticity of substitution measure?**
→ It measures how easily one input (capital or labor) can be substituted for another while maintaining the same output.

8. **What happens when the elasticity of substitution is greater than 1?**
→ It means inputs are **easily substitutable**.

9. **What happens when the elasticity of substitution is less than 1?**
→ It means inputs are **difficult to substitute**.

10. **What does the expansion path show?**
→ It shows the **cost-minimizing combination of inputs** for different levels of output.

11. **What is the equation for the Marginal Rate of Technical Substitution (MRTS)?**
→ $MRTS = \frac{MP_K}{MP_L}$, where $MP_K$ and $MP_L$ are marginal products of capital and labor, respectively.

12. **What is a linear expansion path?**
→ A straight-line expansion path that occurs in a **homothetic production function**.

13. **What is a cost function?**
→ A function that shows the **minimum cost required to produce a given output**.

14. **What are the components of the total cost function?**
→ Fixed costs (FC) and variable costs (VC): $TC = FC + VC$.

15. **What does the term 'returns to scale' refer to?**
→ It refers to how output changes when all inputs are increased proportionally.

16. **What does a decreasing return to scale (DRS) function imply?**
→ Increasing all inputs leads to a **less than proportional increase** in output.

17. **What is the main assumption of a short-run production function?**
→ At least one input is **fixed**, while others are variable.

18. **What does a long-run production function assume?**
→ All inputs are **variable** and can be changed.

19. **What does the term 'marginal cost' refer to?**
→ The additional cost incurred in producing **one more unit** of output.

20. **What is an isoquant?**
→ A curve representing different combinations of inputs that produce the **same level of output**.

21. **What does the price elasticity of demand for inputs determine?**
→ It determines how the **demand for an input** change with a change in its **price**.

22. **What is the significance of cost minimization in production?**
→ It helps firms **maximize profits** by choosing the most efficient combination of inputs.

TWO MARK:

1. **What is the difference between a homogeneous and a homothetic production function?**
→ A homogeneous production function exhibits proportional output changes when all inputs are scaled by the same factor. If the degree of homogeneity is one, it represents constant returns to scale. A homothetic production function, on the other hand, maintains the same marginal rate of technical substitution (MRTS) along rays from the origin, meaning the expansion path remains linear, but it may not be homogeneous.

2. **Explain the significance of the Cobb-Douglas production function.**
→ The Cobb-Douglas production function is widely used to represent production processes where inputs like labor and capital contribute to output. Its general form is $Q = AK^{\alpha}L^{\beta}$, where $A$ represents technology and $\alpha, \beta$ indicate the output elasticities of capital and labor. It is useful in analyzing returns to scale, factor substitutability, and output elasticity.

3. **What is a CES production function and how is it different from Cobb-Douglas?**
→ The CES (Constant Elasticity of Substitution) production function allows flexibility in the substitution between inputs. Unlike the Cobb-Douglas function, where the elasticity of substitution is always 1, the CES function allows for varying degrees of substitutability between capital and labor, enabling better representation of different production processes.

4. **What is elasticity of factor substitution?**
→ Elasticity of factor substitution measures the ease with which one input (e.g., labor) can be replaced with another (e.g., capital) without affecting output. It is defined as the percentage change in

the capital-labor ratio due to a percentage change in the MRTS. A higher elasticity indicates greater substitutability between inputs.

5. **What happens if the elasticity of substitution is greater than 1 or less than 1?**
→ If elasticity is greater than 1, capital and labor are easily substitutable, meaning firms can shift between the two based on cost efficiency. If it is less than 1, substitutability is low, and firms face difficulty in replacing one input with another without affecting output levels.

6. **What is the economic interpretation of an expansion path?**
→ An expansion path represents the cost-minimizing combination of inputs as output levels increase. In the case of constant returns to scale, it is a straight line, while under variable returns, it can be curved. The expansion path helps firms determine how to allocate resources efficiently to increase production.

7. **How is a cost function derived from a production function?**
→ A cost function is derived by determining the minimum cost required to produce a given level of output, given input prices and the production function. Using the firm's optimal input choice, cost functions incorporate fixed and variable costs, leading to the total cost function: TC=FC+VC.

8. **Differentiate between fixed and variable costs.**
→ Fixed costs do not change with output and remain constant in the short run (e.g., rent, salaries of permanent employees). Variable costs change with output level and increase as production rises (e.g., raw materials, wages of temporary workers).

9. **What is the relationship between marginal cost and average cost?**
→ Marginal cost (MC) is the cost of producing one additional unit of output. When MC is less than the average cost (AC), AC decreases, and when MC is greater than AC, AC increases. The MC curve always intersects the AC curve at its minimum point.

10. **What is the short-run production function?**
→ In the short run, at least one input (typically capital) is fixed, while others (like labor) vary. The production function follows the law of diminishing returns, where adding more of a variable input eventually results in decreasing marginal output.

11. **What is the long-run production function?**
→ In the long run, all inputs are variable, allowing firms to adjust production factors fully. The production function in this case helps analyze returns to scale—whether increasing, constant, or decreasing.

12. **Explain the concept of marginal rate of technical substitution (MRTS).**

→ MRTS represents the rate at which one input (e.g., labor) can be substituted for another (e.g., capital) while maintaining the same output level. Mathematically, it is given by the ratio of marginal products: $MRTS = MP_K / MP_L$.

13. **What are returns to scale?**
→ Returns to scale refer to how output responds when all inputs are increased proportionally. If output increases more than proportionally, there are increasing returns to scale; if it increases proportionally, there are constant returns; and if it increases less than proportionally, there are decreasing returns.

14. **Explain the concept of an isoquant.**
→ An isoquant is a curve representing different combinations of inputs that yield the same level of

output. Similar to an indifference curve in consumer theory, an isoquant helps determine the optimal input mix for production.

15. **What is the law of diminishing marginal returns?**
→ This law states that if one input is increased while others remain constant, the additional output generated from the increased input will eventually decline. It explains why firms experience rising marginal costs in the short run.

16. **What is meant by economies of scale?**
→ Economies of scale occur when a firm's long-run average cost decreases as production increases. This happens due to factors like specialization, bulk purchasing, and improved efficiency.

17. **What are diseconomies of scale?**
→ Diseconomies of scale occur when a firm's long-run average cost increases as production expands. This can be due to management inefficiencies, higher coordination costs, or resource limitations.

18. **What is the difference between economic cost and accounting cost?**
→ Economic cost includes both explicit (monetary) costs and implicit (opportunity) costs, while accounting cost considers only explicit costs recorded in financial statements.

19. **Explain the concept of sunk costs.**
→ Sunk costs are costs that have already been incurred and cannot be recovered. These costs should not influence future business decisions.

20. **What is the shutdown point for a firm?**
→ The shutdown point is when a firm's revenue is equal to its variable cost. If price falls below AVC, the firm minimizes losses by ceasing operations.

21. **Explain the significance of isocost lines.**
→ Isocost lines represent combinations of inputs that cost the same amount. They help firms determine the least-cost combination of inputs for given production levels.

22 **How does technological progress affect production functions?**
→ Technological progress shifts the production function upward, allowing firms to produce more output with the same inputs, reducing costs and increasing efficiency.

23. **What is the difference between short-run and long-run costs?**
→ In the short run, some costs are fixed, while others vary with output. In the long run, all costs become variable, allowing firms to adjust production fully.

24. **Explain the concept of total cost, average cost, and marginal cost.**
→ Total cost (TC) is the sum of fixed and variable costs. Average cost (AC) is total cost per unit of output. Marginal cost (MC) is the additional cost of producing one more unit.

25. **What is the L-shaped long-run average cost curve?**
→ The L-shaped LRAC curve suggests that after economies of scale are exhausted, costs remain constant rather than rising significantly due to modern production efficiencies.

26. **What does a firm's supply curve represent?**
→ A firm's supply curve shows the relationship between price and quantity supplied, assuming other factors remain constant.

27. **What is the relationship between cost minimization and profit maximization?**
→ Cost minimization is a necessary condition for profit maximization. A firm must choose input combinations that minimize costs to maximize its profits.

28. **How do firms decide their optimal input combination?**

→ Firms use the least-cost combination rule, where the ratio of input prices equals the MRTS: $MRTS = w/r$, where $w$ is the wage rate and $r$ is the cost of capital.

29. **What is a learning curve in production?**
→ A learning curve shows how labor efficiency improves as workers gain experience, reducing production costs over time.

30. **What is the importance of cost analysis in business decision-making?**
→ Cost analysis helps firms determine pricing, production levels, and profitability, ensuring efficient resource allocation and competitive advantage.

BOARD ANSWER:

**Homothetic Production Functions of a Firm**

The broad class of monotonic increasing functions of homogeneous production functions, which includes also the underlying homogeneous functions, is called homothetic. If the production function is homogeneous (of any degree), the firm's isoclines including long-run expansion path would be straight lines from the origin.

The homothetic production function has the same isoquants as those of its underlying homogeneous function, although, generally, with different quantity indexes.

That is why the firm's expansion path and its isoclines would be straight lines from the origin also for a homothetic production function, and along any such straight line with a fixed ratio of the inputs, the firm's MRTS of L for K or the ratio of $MP_L$ to $MP_K$ would be constant.

This is because for the underlying homogeneous function as also for the monotonic transformations of that function, the MRTS is a function of the ratio of the input quantities. In other words, the ratio of $MP_L$ to $MP_K$ would depend not upon absolute, but upon relative, input quantities.

Therefore, in Fig. 8.26, the homothetic production function would give us

Slope of $IQ_1$ at $A_1$ = Slope of $IQ_2$ at $A_2$ and

Slope of $IQ_1$ at $B_1$ = Slope of $IQ_2$ at $B_2$.

where $A_1$, $A_2$ and $B_1$, $B_2$ are points on two different rays from the origin. That is, the slope of the IQs along any particular straight line from the origin would be a constant. Now, if the slopes of IQs are equal along any ray, then, at any point in the input space, $MP_L/MP_K$ must not change with a proportionate change in L and K.

Looking from the other side, since the input price ratio is constant, the iso-cost lines (ICLs) for different cost levels are parallel. Therefore, at the points of tangency between the ICLs and IQs, the slope of the IQs or the MRTS or $MP_L/MP_K$ would be a constant, being equal to the slope of the ICLs.

This implies that if the production function is to be homothetic, then the ratio of the input quantities would be a constant at the points of tangency, i.e., the points of tangency lie on a ray from the origin. In other words, homotheticity requires that the firm's expansion path coincides with such a ray.

A homogeneous production function is also homothetic—rather, it is a special case of homothetic production functions. In Fig. 8.26, the production function is homogeneous if, in addition, we have $f(tL, tK) = t^nQ$ where t is any positive real number, and n is the degree of homogeneity.

It follows from above that any homogeneous function is a homothetic function, but any homothetic function is not a homogeneous function. For example, $Q = f(L, K) = a$ —$(1/L^{\alpha}K)$ is a homothetic function for it gives us $f_L/f_K = \alpha K/L$ = constant. But it is not a homogeneous function for it does not

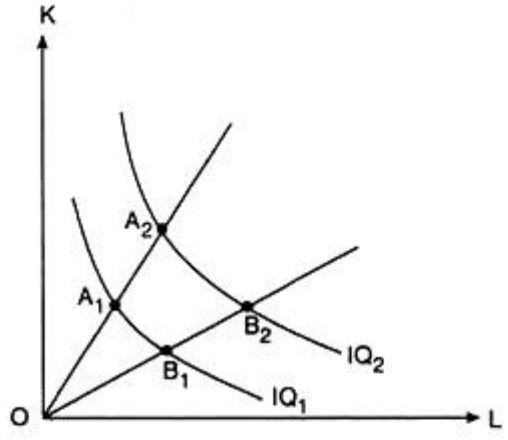



**Fig. 8.26** The IQs under homothetic production function

give us $f(tL, tK) = t^nQ$.

## The CES Production Function:

Arrow, Chenery, Minhas and Solow in their new famous paper of 1961 developed the Constant Elasticity of Substitution (CES) function. This function consists of three variables Q, C and L, and three parameters A, and.

**It may be expressed in the form:**

$Q = A\,[aC^{-\theta} + (1-\alpha)L^{-\theta}]^{-1/\theta}$

where Q is the total output, C is capital, and L is labour. A is the efficiency parameter indicating the state of technology and organisational aspects of production.

It shows that with technological and/or organisational changes, the efficiency parameter leads to a shift in the production function, α (alpha) is the distribution parameter or capital intensity factor coefficient concerned with the relative factor shares in the total output, and θ (theta) is the substitution parameter which determines the elasticity of substitution.

And $A > 0$; $0 < a < 1$; $> -1$.

**Its Properties:**

**The CES production function possesses the following properties:**

1. The CES function is homogenous of degree one. If we increase the inputs C and L in the CES function by n-fold, output Q will also increase by n-fold.

Thus like the Cobb-Douglas production function, the CES function displays constant returns to scale.

2. In the CES production function, the average and marginal products in the variables C and L are homogeneous of degree zero like all linearly homogeneous production functions.

3. From the above property, the slope of an isoquant, i.e., the MRTS of capital for labour can be shown to be convex to the origin.

4. The parameter (theta) in the CES production function determines the elasticity of substitution. In this function, the elasticity of substitution,

$\sigma = 1/ 1 + \theta$

This shows that he elasticity of substitution is a constant whose magnitude depends on the value of the parameter θ. If α =0, then a = 1. If θ = ∞, then a =0. If θ = -0, then a =∞. This reveals that when a = 1, the CES production function becomes the Cobb-Douglas production function.

If Q < 0, then a = – 0; and if в < ∞, then a < 1. Thus the isoquants for the CES production function range from right angles to straight lines as the elasticity substitution ranges from 0 to.

5. As a corollary of the above, if L and C inputs are substitutable ∞ for each other an increase in C will require less of L for a given output. As a result, the MP of L will increase. Thus, the MP of an input will increase when the other input is increased.

**Merits of C.E.S. Production Function:**

**The CES function has the following merits:**

1. CES function is more general.

2. CES function covers all types of returns.

3. CES function takes account of a number of parameters.

4. CES function takes account of raw materials among its inputs.

5. CES function is very easy to estimate.

ADVERTISEMENTS:

6. CES function is free from unrealistic assumptions.

**CES function vs. CD function:**

**There are some fundamental differences between the CES function and the CD production function:**

1. The CD function is based on the observation that the wage rate is a constant proportion of output per head. On the other hand, the CES function is based on the observation that output per head is a changing proportion of wage rate.

2. The CES production function is based on larger parameters than the CD production function and as such allows factors to be either substitutes or complements. The CD function is, on the other hand, based on the assumption of substitutability of factors and neglects the complementarity of factors. Thus the CES function has wider scope and applicability.

3. The CES production function can be extended to more than two inputs, unlike the CD function which is applicable to only two inputs.

4. In the CES function, the elasticity of substitution is constant but not necessarily equal to unity. It ranges from 0 to ∞. But the CD function is related to elasticity equal to unity. Thus the CD function is a special case of the CES function.

5. The CES function covers constant, increasing and decreasing returns to scale, while the CD function relates to only constant returns to scale.

**Limitations of CES Production Function:**

**But the CES function has certain limitations:**

1. The CES production function considers only two inputs. It can be extended to more than two inputs. But it becomes very difficult and complicated mathematically to use it for more than two inputs.

2. The distribution parameter or capital intensity factor coefficient, α is not dimensionless.

3. If data are fitted to the CES function, the value of the efficiency parameter A cannot be made independent of 0 or of the units of Q, C and L.

4. If the CES function is used to describe the production function of a firm, it cannot be used to describe the aggregate production function of all the firms in the industry. Thus it involves the problem of aggregation of production function of different firms in the industry.

5. It suffers from the drawback that elasticity of substitution between any part of inputs is the same which does not appear to be realistic.

6. In estimating the parameters of CES production function, we may encounter a large number of problems like choice of exogenous variables, estimation procedure and the problem of multi-collinear ties.

## The Cobb-Douglas Production Function:

The Cobb-Douglas production function is based on the empirical study of the American manufacturing industry made by Paul H. Douglas and C.W. Cobb. It is a linear homogeneous production function of degree one which takes into account two inputs, labour and capital, for the entire output of the. manufacturing industry.

**The Cobb-Douglas production function is expressed as:**

$Q = AL^{a} C^{\beta}$

where Q is output and L and C are inputs of labour and capital respectively. A, a and β are positive parameters where = a > O, β > O.

The equation tells that output depends directly on L and C, and that part of output which cannot be explained by L and C is explained by A which is the 'residual', often called technical change.

The production function solved by Cobb-Douglas had 1/4 contribution of capital to the increase in manufacturing industry and 3/4 of labour so that the C-D production function is

$Q = AL^{3/4} C1^{/4}$

which shows constant returns to scale because the total of the values of L and C is equal to one: (3/4 + 1/4), i.e.,(a + β = 1) . The coefficient of labourer in the C-D function measures the percentage increase in (Q that would result from a 1 per cent increase in L, while holding C as constant.

Similarly, B is the percentage increase in Q that would result from a 1 per cent increase in C, while holding L as constant. The C-D production function showing constant returns to scale is depicted in Figure 20. Labour input is taken on the horizontal axis and capital on the vertical axis.

To produce 100 units of output, OC, units of capital and OL units of labour are used. If the output were to be doubled to 200, the inputs of labour and capital would have to be doubled. OC is exactly double of $OC_1$ and of $OL_2$ is double of $OL_2$.

Similarly, if the output is to be raised three-fold to 300, the units of labour and capital will have to be increased three-fold. $OC_3$ and $OL_3$ are three times larger than $OC_1$, and $OL_1$, respectively. Another method is to take the scale line or expansion path connecting the equilibrium points Q, P and R. OS is the scale line or expansion path joining these points.

It shows that the isoquants 100, 200 and 300 are equidistant. Thus, on the OS scale line OQ = QP = PR which shows that when capital and labour are increased in equal proportions, the output also increases in the same proportion.

**Criticisms of C-D Production Function:**

**The C-D production function has been criticised by Arrow, Chenery, Minhas and Solow as discussed below:**

1. The C-D production function considers only two inputs, labour and capital, and neglects some important inputs, like raw materials, which are used in production. It is, therefore, not possible to generalize this function to more than two inputs.

2. In the C-D production function, the problem of measurement of capital arises because it takes only the quantity of capital available for production. But the full use of the available capital can be made only in periods of full employment. This is unrealistic because no economy is always fully employed.

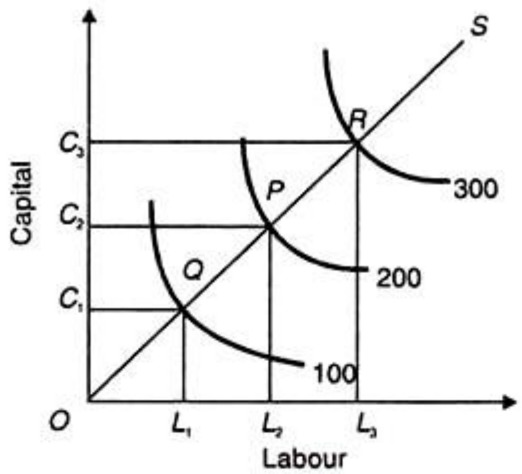



**Fig. 20**

3. The C-D production function is criticised because it shows constant returns to scale. But constant returns to scale are not an actuality, for either increasing or decreasing returns to scale are applicable to production.

It is not possible to change all inputs to bring a proportionate change in the outputs of all the industries. Some inputs are scarce and cannot be increased in the same proportion as abundant inputs. On the other hand, inputs like machines, entrepreneurship, etc. are indivisible. As output increases due to the use of indivisible factors to their maximum capacity, per unit cost falls.

Thus when the supply of inputs is scarce and indivisibilities are present, constant returns to scale are not possible. Whenever the units of different inputs are increased in the production process, economies of scale and specialization lead to increasing returns to scale.

In practice, however, no entrepreneur will like to increase the various units of inputs in order to have a proportionate increase in output. His endeavour is to have more than proportionate increase in output, though diminishing returns to scale are also not ruled out.

4. The C-D production function is based on the assumption of substitutability of factors and neglects the complementarity of factors.

5. This function is based on the assumption of perfect competition in the factor market which is unrealistic. If, however, this assumption is dropped, the coefficients $\alpha$ and $\beta$ do not represent factor shares.

6. One of the weaknesses of C-D function is the aggregation problem. This problem arises when this function is applied to every firm in an industry and to the entire industry. In this situation, there will be many production functions of low or high aggregation. Thus the C-D function does not measure what it aims at measuring.

**Conclusion:**

Thus the practicability of the C-D production function in the manufacturing industry is a doubtful proposition. This is not applicable to agriculture where for intensive cultivation, increasing the quantities of inputs will not raise output proportionately. Even then, it cannot be denied that constant returns to scale are a stage in the life of a firm, industry or economy. It is another thing that this stage may come after some time and for a short while.

**It's Importance:**

Despite these criticisms, the C-D function is of much importance.

1. It has been used widely in empirical studies of manufacturing industries and in inter-industry comparisons.

2. It is used to determine the relative shares of labour and capital in total output.

3. It is used to prove Euler's Theorem.

4. Its parameters a and b represent elasticity coefficients that are used for inter-sectoral comparisons.

5. This production function is linear homogeneous of degree one which shows constant returns to scale, If $\alpha + \beta = 1$, there are increasing returns to scale and if $\alpha + \beta < 1$, there are diminishing returns to scale.

6. Economists have extended this production function to more than two variables.

## Derivation of Cost Functions from Production Functions:

Costs are derived functions. They are derived from the technological relationships implied by the production function.

We will first show how to derive graphically the cost curves from the production function. Subsequently we will derive mathematically the total-cost function from a Cobb-Douglas production function.

**A. Graphical Derivation of Cost Curves from the Production Function:**

The total cost curve is determined by the locus of points of tangency of successive iso-cost lines with higher isoquants.

**Assumptions for our example:**

(a) Given production function (that is, constant technology) with constant returns to scale;

(b) Given prices of factors

w = 20p per man hour

r = 20p per machine hour

The following methods of production are part of the available technology of the firm. They refer to the quantities of L and K required for the production of one ton of output which is the 'unit' level.

| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ |
|---|---|---|---|---|---|---|---|---|
| Labour: hours | 2 | 3·0 | 4·0 | 5·0 | 6·0 | 7·0 | 8·0 | 9 |
| Capital: hours | 6 | 4·5 | 4·0 | 3·7 | 3·5 | 3·3 | 3·1 | 3 |

**The cost of each method for the production of one 'unit' of output (given the above factor prices) is as follows:**

| | Cost $P_1$ | Cost $P_2$ | Cost $P_3$ | Cost $P_4$ | Cost $P_5$ | Cost $P_6$ | Cost $P_7$ | Cost $P_8$ |
|---|---|---|---|---|---|---|---|---|
| Labour cost | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 |
| Capital cost | 120 | 90 | 80 | 74 | 70 | 66 | 62 | 60 |
| Total cost | 160 | 150 | 160 | 174 | 190 | 206 | 222 | 240 |

Clearly the least-cost method of production, given our assumptions, is the second method ($P_2$). This method will be chosen by the rational entrepreneur for all levels of output (given the assumption of constant returns to scale). Table 3.1 includes some levels of output and their respective total costs (for the chosen least-cost method of production, $P_2$). The product expansion path is shown in figure 3.42. It is formed from the points of tangency of the iso-costs and the isoquants. The TC curve may be drawn from the information (on output and costs) provided by the points of tangency. For example,

| | | |
|---|---|---|
| at point *a*, | $X = 5$ | $TC = 750$ |
| at point *b* | $X = 10$ | $TC = 1{,}500$ |
| at point *d* | $X = 20$ | $TC = 3{,}000$ etc. |

Table 3.1 Output levels and *TC* (for activity $P_2$)

| *Output X* (in tons) | *Total cost* (in pence) | *AC* (pence per ton) |
|---|---|---|
| 0 | 0 | – |
| 5 | 750 | 150 |
| 10 | 1,500 | 150 |
| 15 | 2,250 | 150 |
| 20 | 3,000 | 150 |
| 25 | 3,750 | 150 |
| 30 | 4,500 | 150 |
| 35 | 5,250 | 150 |
| 40 | 6,000 | 150 |
| 45 | 6,750 | 150 |
| 50 | 7,500 | 150 |
| 55 | 8,250 | 150 |
| 60 | 9,000 | 150 |
| 65 | 9,750 | 150 |
| 70 | 10,500 | 150 |

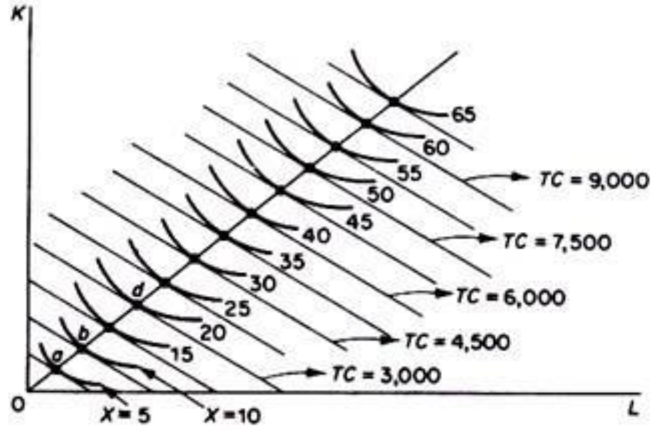



Figure 3.42

Plotting these points on a two-dimensional diagram with TC on the vertical axis and output (X) on the horizontal axis, we obtain the total-cost curve (figure 3.43). With our assumption (of constant returns to scale and of constant factor prices) the AC is constant (£1.50 per 'unit' of output), hence the AC will be a straight line, parallel to the horizontal axis (figure 3.44). It is important to remember that the cost curves assume that the problem of choice of the optimal (least-cost) technique has been solved at a previous stage. In other words, the complex problem of finding the cheapest combination of factor inputs must be solved before the cost curve is defined.

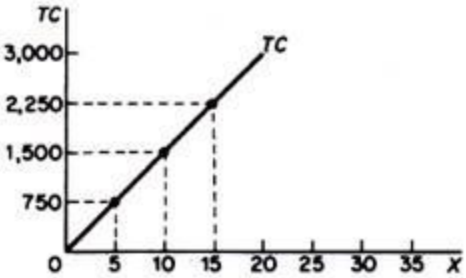



Figure 3.43

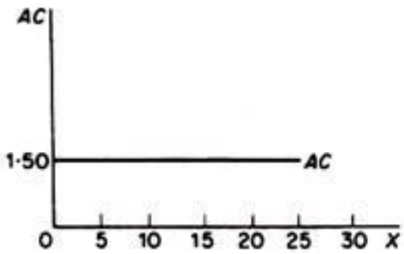



Figure 3.44

**B. Formal Derivation of Cost Curves from a Production Function:**

In applied research one of the most commonly used forms of production function is the Cobb-Douglas form

$$X = b_0 L^{b_1} K^{b_2}$$

Given this production function and the cost equation

$$C = wL + rK$$

we want to derive the cost function, that is, the total cost as a function of output

$$C = f(X)$$

We begin by solving the constrained output maximisation problem:

Maximise $\quad X = b_0 L^{b_1} K^{b_2}$

subject to $\quad \bar{C} = wL + rK \quad$ (cost constraint)

(The bar on top of $C$ has the meaning that the firm has a *given* amount of money to spend on both factors of production.)
We form the 'composite' function

$$\phi = X + \lambda(\bar{C} - wL - rK)$$

where $\lambda$ = Lagrangian multiplier

The first condition for maximisation is that the first derivatives of the function with respect to $L$, $K$ and $\lambda$ be equal to zero:

$$\frac{\partial \phi}{\partial L} = b_1 \frac{X}{L} - \lambda w = 0 \tag{3.4}$$

$$\frac{\partial \phi}{\partial K} = b_2 \frac{X}{K} - \lambda r = 0 \tag{3.5}$$

$$\frac{\partial \phi}{\partial \lambda} = (C - wL - rK) = 0 \tag{3.6}$$

From equations 3.4 and 3.5 we obtain

$$b_1 \frac{X}{L} = \lambda w \quad \text{and} \quad b_2 \frac{X}{K} = \lambda r$$

Dividing these expressions we obtain

$$\frac{b_1}{b_2} \cdot \frac{K}{L} = \frac{w}{r}$$

Solving for $K$

$$K = \frac{w}{r} \cdot \frac{b_2}{b_1} L \tag{3.7}$$

Substituting $K$ into the production function we obtain

$$X = b_0 L^{b_1} K^{b_2}$$

$$X = b_0 L^{b_1} \left[\frac{w}{r} \frac{b_2}{b_1} L\right]^{b_2}$$

$$X = b_0 \left[\left(\frac{w}{r}\right)\left(\frac{b_2}{b_1}\right)\right]^{b_2} L^{(b_1 + b_2)}$$

The term in brackets is the constant term of the function: it includes the three coefficients of the production function, $b_0$, $b_1$, $b_2$, and the prices of the factors of production.

Solving the above form of the production function for $L$, we find

$$\frac{1}{b_0 \left(\frac{w}{r} \frac{b_2}{b_1}\right)^{b_2}} X = L^{(b_1 + b_2)}$$

or

$$\left[\frac{1}{b_0 \left(\frac{w}{r} \frac{b_2}{b_1}\right)^{b_2}} \cdot X\right]^{1/(b_1 + b_2)} = L$$

or

$$L = \left(\frac{rb_1}{wb_2}\right)^{b_2/(b_1 + b_2)} \left(\frac{X}{b_0}\right)^{1/(b_1 + b_2)} \tag{3.8}$$

Substituting the value of $L$ from expression 3.8 into expression 3.7 for capital we obtain

$$K = \frac{w}{r} \cdot \frac{b_2}{b_1} L$$

$$K = \left(\frac{wb_2}{rb_1}\right)^{1/(b_1 + b_2)} \left(\frac{X}{b_0}\right)^{1/(b_1 + b_2)} \tag{3.9}$$

Substituting expression 3.8 and 3.9 into the cost equation $C = wL + rK$ we find

$$C = \left(\frac{1}{b_0}\right)^{1/(b_1 + b_2)} \left[w\left(\frac{rb_1}{wb_2}\right)^{b_2/(b_1 + b_2)} + r\left(\frac{wb_2}{rb_1}\right)^{b_1/(b_1 + b_2)}\right] \cdot X^{1/(b_1 + b_2)}$$

**Rearranging the expression above we obtain:**

$$C = \left\{\left(\frac{1}{b_0}\right)\left[\left(\frac{b_1}{b_2}\right)^{b_2} + \left(\frac{b_2}{b_1}\right)^{b_1}\right]\right\}^{1/(b_1 + b_2)} \cdot \left\{w^{b_1/(b_1 + b_2)} \cdot r^{b_2/(b_1 + b_2)}\right\} \cdot X^{1/(b_1 + b_2)}$$

**This is the cost function, that is, the cost expressed as a function of:**

ADVERTISEMENTS:

(i) Output, X;

(ii) The production function coefficients, $b_0$, $b_1$, $b_2$; (clearly the sum $b_1 + b_2$ is a measure of the returns to scale);

(iii) The prices of factors, w, r.

If prices of factors are given (the usual assumption in the theory of the firm), cost depends only on output x, and we can draw the usual diagrams of cost curves, which express graphically the cost function

C = f(X) ceteris paribus

'Ceteris paribus' implies that all other determinants of costs, that is, the production technology and the prices of factors, remain unchanged. If these factors change, the cost curve will shift upwards or downwards.

UNIT – 3

THEORY:

UNIT – 5

THEORY:

**Introduction to Welfare Economics**

Welfare economics is a branch of economics that focuses on evaluating the well-being of individuals and society as a whole. It assesses how different economic policies, market structures, and resource allocations affect social welfare. The primary objective is to determine conditions under which economic activities lead to the maximization of societal well-being.

Welfare economics utilizes various theoretical criteria to measure social welfare, including the Pigouvian welfare criterion, Pareto optimality, the Kaldor-Hicks compensation criterion, and the concept of a social welfare function. These approaches help policymakers analyze whether economic changes lead to improvements in overall societal welfare.

---

**Nature of Welfare Economics**

Welfare economics is concerned with the normative aspect of economics, meaning it evaluates economic outcomes based on value judgments rather than just factual analysis. It studies:

1. **Efficiency and Equity** – How resources are allocated efficiently while considering fair distribution.
2. **Social Welfare Maximization** – The best possible outcomes for society as a whole.
3. **Market Failures and Government Intervention** – Identifying situations where markets fail to achieve efficiency and the role of government policies in correcting them.

Welfare economics is built on two primary theorems:

- **First Fundamental Theorem of Welfare Economics**: If markets are perfectly competitive and free from externalities, the equilibrium outcome is Pareto efficient.
- **Second Fundamental Theorem of Welfare Economics**: Any desired efficient allocation can be achieved through appropriate redistribution of resources before market transactions take place.

---

**Pigouvian Welfare Criterion**

A.C. Pigou, in his book *The Economics of Welfare (1920)*, proposed that economic welfare depends on the sum total of individual utilities. He introduced the concept of **externalities**—costs or benefits imposed on third parties that are not reflected in market prices.

**Key Points of Pigouvian Welfare Economics**

- **Positive Externalities**: Benefits that spill over to others, such as education and public health initiatives.
- **Negative Externalities**: Costs imposed on others, such as pollution and traffic congestion.
- **Government Intervention**: To correct market failures, Pigou suggested imposing **Pigouvian taxes** on negative externalities (e.g., carbon taxes) and providing subsidies for positive externalities.

Pigou's welfare criterion argues that state intervention can improve economic welfare by addressing externalities through taxation, subsidies, or regulation.

---

**Pareto Optimality Criterion**

Named after **Vilfredo Pareto**, this criterion states that an economic allocation is **Pareto efficient** if no one can be made better off without making someone else worse off.

**Conditions for Pareto Optimality**

For an economy to be Pareto optimal, three efficiency conditions must be met:

1. **Efficiency in Exchange (Consumption Efficiency)**: Goods should be distributed among consumers in such a way that their marginal rate of substitution (MRS) is equal.
   - Mathematically: **MRS_A = MRS_B**
2. **Efficiency in Production (Production Efficiency)**: Inputs should be allocated in such a way that the marginal rate of technical substitution (MRTS) is equal across all firms.
   - Mathematically: **MRTS_KL (Firm 1) = MRTS_KL (Firm 2)**
3. **Efficiency in Product Mix (Overall Efficiency)**: The economy's resource allocation should align with consumer preferences, meaning the marginal rate of transformation (MRT) should equal the marginal rate of substitution (MRS).
   - Mathematically: **MRS = MRT**

**Limitations of Pareto Efficiency**

- It does not consider fairness or income distribution.
- A Pareto-efficient allocation may still be highly unequal.
- It does not provide a mechanism to move from one Pareto-efficient state to another.

---

**Kaldor-Hicks Compensation Criterion**

The **Kaldor-Hicks criterion** (developed by Nicholas Kaldor and John Hicks) is a broader measure of welfare changes in an economy. Unlike Pareto optimality, it allows for welfare improvements even if some individuals are worse off, as long as the winners can compensate the losers.

**Key Aspects of Kaldor-Hicks Efficiency**

- **Compensation Principle**: If a policy change benefits some individuals so much that they could, in theory, compensate the losers and still be better off, the policy is considered an improvement.
- **Potential Pareto Improvement**: Even if actual compensation does not occur, an outcome is still considered efficient if such compensation is possible in theory.

**Limitations of Kaldor-Hicks Criterion**

- In practice, compensation is rarely implemented, leading to potential unfairness.
- It does not necessarily lead to an equitable distribution of resources.
- It may justify policies that increase inequality if they lead to overall efficiency.

**Social Welfare Function**

The **social welfare function (SWF)** is a mathematical tool used to aggregate individual utilities into a single measure of societal welfare. Developed by **Bergson and Samuelson**, it provides a framework for evaluating different economic states based on societal preferences.

**Types of Social Welfare Functions**

1. **Utilitarian SWF (Benthamite Approach):**
   - **Formula:** $W = U_1 + U_2 + ... + U_n$
   - Suggests that the best economic outcome is the one that maximizes total utility, regardless of distribution.
   - Criticized for ignoring inequality.
2. **Rawlsian SWF (Maximin Principle):**
   - **Formula:** $W = \min(U_1, U_2, ..., U_n)$
   - Based on John Rawls' *Theory of Justice*, this approach seeks to maximize the utility of the worst-off individual in society.
   - Prioritizes fairness over total welfare.

4. **Bergson-Samuelson SWF**:
   - Allows for different weightings of individual utilities, offering flexibility in welfare analysis.
5. **Arrow's Impossibility Theorem**:
   - Kenneth Arrow proved that no social welfare function can simultaneously satisfy all fairness criteria (unrestricted domain, non-dictatorship, Pareto efficiency, independence of irrelevant alternatives).
   - This suggests that designing a perfect measure of social welfare is mathematically impossible.

ONE MARK:

1. **What is Welfare Economics?**
   - Welfare Economics is the study of how economic policies and resource allocation affect social well-being.
2. **Who introduced the concept of externalities in Welfare Economics?**
   - A.C. Pigou introduced the concept of externalities.
3. **What is the primary goal of Welfare Economics?**
   - The primary goal is to maximize social welfare through efficient resource allocation and fair distribution.
4. **What are the two main theorems of Welfare Economics?**
   - The First and Second Fundamental Theorems of Welfare Economics.

5. **State the First Fundamental Theorem of Welfare Economics.**
    - It states that a perfectly competitive market leads to a Pareto-efficient outcome.
6. **What does Pareto Optimality mean?**
    - It means an allocation where no one can be made better off without making someone else worse off.
7. **Who introduced the concept of Pareto Efficiency?**
    - Vilfredo Pareto.
8. **What is the main limitation of Pareto Efficiency?**
    - It does not consider fairness or income distribution.
9. **What is the Kaldor-Hicks Compensation Principle?**
    - It states that a policy change is beneficial if the winners can compensate the losers and still be better off.
10. **Who developed the Kaldor-Hicks Criterion?**

- Nicholas Kaldor and John Hicks.

11. **What is Pigouvian Tax?**

- A tax imposed on activities that generate negative externalities, such as pollution.

12. **What is the purpose of Pigouvian subsidies?**

- To encourage activities that generate positive externalities, like education.

13. **What is the Social Welfare Function?**

- A mathematical representation of society's collective well-being based on individual utilities.

14. **Who developed the Social Welfare Function?**

- Abram Bergson and Paul Samuelson.

15. **What is a Utilitarian Social Welfare Function?**

- It aims to maximize total social utility by summing up individual utilities.

16. **What is the Rawlsian Social Welfare Function?**

- It prioritizes improving the welfare of the worst-off individual in society.

17. **What does Arrow's Impossibility Theorem state?**

- No social welfare function can satisfy all fairness criteria simultaneously.

18. **Who introduced the concept of Arrow's Impossibility Theorem?**

- Kenneth Arrow.

19. **What is the Efficiency in Exchange condition for Pareto Optimality?**

- It requires that the Marginal Rate of Substitution (MRS) be equal for all consumers.

20. **What is the Efficiency in Production condition for Pareto Optimality?**

- It requires that the Marginal Rate of Technical Substitution (MRTS) be equal for all firms.

21. **What is the role of government in Welfare Economics?**

- To correct market failures, redistribute income, and improve social welfare.

22. **What does the Second Fundamental Theorem of Welfare Economics state?**

- Any Pareto-efficient outcome can be achieved through an appropriate redistribution of resources.

23. **What is meant by Market Failure in Welfare Economics?**

- A situation where the market fails to allocate resources efficiently.

24. **What is a Merit Good?**

- A good that is under-consumed in a free market but has social benefits, such as education.

25. **What is the difference between Positive and Normative Welfare Economics?**

- Positive Welfare Economics describes what is, while Normative Welfare Economics suggests what should be done to improve welfare.

TWO MARK:

1. **What is Welfare Economics?**
Welfare Economics is a branch of economics that focuses on evaluating economic policies and their impact on social well-being. It studies how resources can be allocated to maximize overall societal welfare, considering factors like efficiency and equity.

2. **What are the two fundamental theorems of Welfare Economics?**
The first theorem states that under perfect competition, markets lead to Pareto-efficient outcomes. The second theorem asserts that any Pareto-efficient allocation can be achieved through appropriate redistribution without affecting efficiency.

3. **Explain Pareto Optimality with an example.**
Pareto Optimality occurs when no individual can be made better off without making someone else worse off. For example, if reallocating food from one person to another does not improve overall satisfaction, the distribution is Pareto efficient.

4. **What is the difference between Efficiency and Equity in Welfare Economics?**
Efficiency refers to resource allocation that maximizes total output, while equity concerns fair distribution among individuals. A policy may be efficient but not equitable if wealth is concentrated in a few hands.

5. **What is Pigouvian Welfare Criterion?**
A.C. Pigou’s criterion suggests that government intervention through taxes or subsidies can correct externalities and improve social welfare. For example, imposing a pollution tax can reduce harmful emissions.

6. **What is the significance of Pigouvian taxes?**
Pigouvian taxes are imposed on goods that create negative externalities, such as carbon emissions. They help internalize social costs, making producers and consumers account for the external harm caused by their actions.

7. **Explain the Kaldor-Hicks Compensation Criterion.**
This principle states that a policy change is beneficial if the winners can compensate the losers, even if compensation does not actually occur. It extends Pareto efficiency by allowing redistribution in theory.

8. **How does the Kaldor-Hicks Criterion differ from Pareto Optimality?**
While Pareto Optimality requires no one to be worse off, Kaldor-Hicks allows for losses as long as the overall benefits outweigh the costs. It is often used to justify economic policies with winners and losers.

9. **What is the Social Welfare Function?**
The Social Welfare Function is a mathematical formula that aggregates individual utilities to measure overall social well-being. It helps policymakers evaluate different economic policies based on societal preferences.

10. **Compare Utilitarian and Rawlsian Social Welfare Functions.**
The Utilitarian function maximizes total utility, considering the sum of individual well-being. The Rawlsian function prioritizes the welfare of the worst-off individual, advocating for more equitable distribution.

11. **What is Arrow's Impossibility Theorem?**
Kenneth Arrow's theorem states that no voting system can satisfy all fairness criteria—such as unrestricted domain, non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives—simultaneously in collective decision-making.

12. **How does Market Failure relate to Welfare Economics?**
Market failure occurs when free markets fail to allocate resources efficiently, leading to loss of social welfare. Common causes include externalities, public goods, and monopolies, necessitating government intervention.

13. **What is the role of government in Welfare Economics?**
The government plays a role in correcting market failures, redistributing income, and ensuring social welfare through policies like taxation, subsidies, and public goods provision.

14. **Explain the concept of Externalities in Welfare Economics.**
Externalities occur when the actions of individuals or firms impact others without compensation. Positive externalities, like education, benefit society, while negative externalities, like pollution, impose costs.

15. **What is a Public Good? Give an example.**
A public good is non-rivalrous and non-excludable, meaning one person's consumption does not reduce availability for others, and no one can be prevented from using it. Examples include street lighting and national defense.

16. **What is the meaning of Economic Efficiency in Welfare Economics?**
Economic efficiency is achieved when resources are allocated in a way that maximizes total welfare. This occurs when production, exchange, and consumption are optimized without waste.

17. **What is the Edgeworth Box and its significance in Welfare Economics?**
The Edgeworth Box is a graphical tool used to show efficient resource allocation between two consumers. It helps illustrate Pareto optimal allocations and efficiency in exchange.

18. **What does the Second Fundamental Theorem of Welfare Economics state?**
It states that any Pareto-efficient outcome can be achieved through a suitable redistribution of resources, followed by market operations, ensuring both efficiency and equity.

19. **Explain the concept of Marginal Rate of Substitution (MRS) in Welfare Economics.**
MRS is the rate at which a consumer is willing to substitute one good for another while maintaining the same level of satisfaction. At Pareto efficiency, the MRS of all individuals must be equal.

20. **What is the Marginal Rate of Transformation (MRT) in production?**
MRT is the rate at which one good can be transformed into another by reallocating resources. Efficiency in production requires that the MRT equals the MRS in consumption.

21. **How does Welfare Economics apply to Labour Markets?**
Welfare Economics evaluates policies like minimum wage laws, social security, and taxation to ensure fairness and efficiency in labour markets, balancing worker benefits with economic productivity.

22. **What is the significance of Cost-Benefit Analysis in Welfare Economics?**
Cost-Benefit Analysis helps policymakers assess whether a project or policy improves welfare by comparing total benefits with total costs. Projects with net positive benefits are preferred.

23. **What is the difference between Positive and Normative Welfare Economics?**
Positive Welfare Economics describes what is, analyzing economic behavior objectively, while Normative Welfare Economics suggests what should be, focusing on policy recommendations to improve welfare.

24. **How does Intergenerational Equity relate to Welfare Economics?**
Intergenerational Equity ensures that future generations inherit a fair share of resources. It addresses long-term sustainability issues like environmental conservation and public debt management.

25. **What is the concept of Contract Curve in Welfare Economics?**
The Contract Curve represents all Pareto-efficient allocations in the Edgeworth Box where MRS between two individuals is equal. It shows efficient exchange possibilities in an economy.

26. **Explain the meaning of a Merit Good with an example.**
Merit goods are under-consumed in a free market but provide societal benefits. For example, education improves individual skills and contributes to economic growth, justifying government support.

27. **What are the main criticisms of the Kaldor-Hicks Criterion?**
Critics argue that it ignores fairness since compensation is not mandatory, and it can justify policies that make some individuals significantly worse off, leading to social inequality.

28. **How does Redistribution of Income impact Welfare Economics?**
Redistribution through taxation and social programs helps achieve a more equitable income distribution, reducing poverty and improving social welfare without necessarily harming economic efficiency.

29. **What is meant by the Pareto Improvement concept?**
A Pareto Improvement occurs when at least one individual is made better off without making anyone worse off. It is often used to evaluate economic policy changes.

30. **Why is Welfare Economics important in policy-making?**
Welfare Economics provides a framework for evaluating policies that affect social well-being, helping governments design tax systems, subsidies, and public services to improve overall welfare.

BOARD ANSWER:

## A.C. Pigou's Economic of Welfare:

**Meaning of Welfare:**

According to Pigou, welfare resides in a man's state of mind or consciousness which is made up of his satisfactions or utilities. The basis of welfare, therefore, is necessarily the extent to which an individual's desires are met.

Social welfare is regarded as the summation of all individual welfares in a society. Since general welfare is a very wide, complicated and impracticable notion, Pigou delimits the range of his study to economic welfare. As he himself observes, economic welfare is by no means an index of total welfare because many other elements in the latter, like the quality of work, one's environment, human relationships, status, housing, and public security are absent from economic welfare.

He, therefore, defines economic welfare as **"that part of social (general) welfare that can be brought directly or indirectly into relation with the measuring rod of money."** Thus economic welfare, in the Pigovian sense, implies the satisfaction of utility derived by an individual from the use of exchangeable goods and services.

**Pigovian Welfare Conditions:**

Pigou regard economic welfare and national income as essentially coordinate. It is on this basis that he lays down two conditions for maximisation of welfare. The first condition states that welfare is said to increase when national income increases. Given the same tastes and income distribution, an increase in the national income represents an increase in welfare. Pigou contends that in most cases the national income would increase even though the disutility of work also increases.

Second, for welfare maximisation the distribution of the national income is equally important. If national income remains constant, transfers of income from the rich to the poor would improve welfare. According to Pigou, such transfers mean less to the wealthy than to the poor, as a result the economic position of the latter is raised. This welfare condition is based on the dual Pigovian postulates of 'equal capacity for satisfaction and diminishing marginal utility of income.'

Pigou argues that different people derive the same satisfaction out of the same real income and that **"people now rich are different in kind from the people now poor having in their fundamental nature greater capacities for enjoyment."** With income subject to diminishing marginal utility, transfers of income from the rich to the poor will increase social welfare by satisfying the more intense wants of the latter at the expense of the less intense wants of the former. This it is economic equality that maximises welfare.

**Dual Criterion:**

**To find out improvements in social welfare, Pigou adopts a dual criterion:**

First, an increase in the national income 'brought about either by increasing some goods without dimishing others or by transferring factors to activities in which their social value is higher,' is regarded an improvement in welfare without reducing the share of the poor.

Second, any reorganization of the economy which increases the share of the poor without reducing the national income is also considered an improvement in social welfare.

**Assumptions of Pigovian Conditions:**

**The Pigovian welfare conditions and the dual criterion pre-suppose the existence of the following assumptions:**

(1) Each individual tries to maximise his satisfaction from his expenditure on different goods and services.

(2) Satisfactions are comparable both interpersonally and interpersonally.

(3) The law of diminishing marginal utility of income applies. It means that the marginal utility of income falls, as income increases. As a result, the gain in utility of an additional amount of income to a poor man is greater than the loss of utility to a rich man from the same amount of income.

(4) There is equal capacity for satisfaction. It implies that different people derive the same satisfaction out of the same real income. Given these assumptions, it is possible to satisfy the Pigovian conditions of maximum social welfare on the basis of his dual criterion.

**Its Criticism:**

**Though, Pigou's Economics of Welfare is the first clear analysis of welfare economics, yet the Pigovian 'conditions of welfare' have been criticised on the following grounds:**

**(1) The Notion of Maximisation is not clear:**

Pigou lays emphasis on the maximisation of welfare, but he does not clarify the notion of maximisation. His 'maximum' is in fact the 'optimum', but it is a stable point. But this is not a correct view because the 'optimum' is not stable. It changes with the increase or decrease of national income.

**(2) Pigou measures 'welfare' cardinally:**

According to Pigou, welfare is measured in terms of utility or satisfaction. He regards social welfare as the summation of utilities of exchangeable goods and services to individuals. Economists do not agree with this view because quantitative measurement of utility is not possible. It is for this reason that modern economist's measure utility ordinally.

**(3) National Income is not an Accurate Measure of Welfare:**

Pigou's welfare conditions are related to the national income. But it is not easy to calculate national income. Again, social welfare does not increase by a mere increase in national income. It is possible that national income may increase due to inflationary rise in prices and the poor may become worse-off than before.

It is due to these reasons that modern economists measure welfare on the basis of 'choice' rather than by national income. For instance, when an individual chooses bundle A of some good rather than bundle 5, he undoubtedly derives greater satisfaction or utility from A. It is in this way that his welfare increases.

**(4)** According to Professor Robbins, Pigou's assumption of man's equal capacity for satisfaction does not make his notion of welfare a positive study. In his words, "This assumption rests on ethical principle rather than upon scientific demonstration; it is not a judgement of value."

**(5) Pigou does not clarify the ethical relation of welfare:**

Welfare economics is closely related to ethics but Pigou does not clarify it. Welfare economics is essentially a normative study in which value judgements and interpersonal comparisons are made. By not relating these concepts with his notion of welfare, Pigou s economics of welfare is not considered as an objective study of the causes of welfare.

# THE PARETO-OPTIMALITY CRITERION

This criterion refers to economic efficiency which can be objectively measured. It is called Pareto criterion after the famous Italian economist **Vilfredo Pareto (1848-1923)**. According to this criterion any change that makes at least one individual better-off and no one worse-off is an improvement in social welfare. Conversely, a change that makes no one better-off and at least one worse-off is a decrease in social welfare.

The criterion can be stated in a somewhat different way: a situation in which it is impossible to make anyone better-off without making someone worse-off is said to be Pareto-optimal or Pareto-efficient.

For the attainment of a Pareto-efficient situation in an economy three marginal conditions must be satisfied: *(a) Efficiency of distribution of commodities among consumers (efficiency in exchange); (b) Efficiency of the allocation of factors among firms (efficiency of production); (c) Efficiency in the allocation of factors among commodities (efficiency in the product-mix, or composition of output).*

Before exa- mining these marginal conditions we discuss briefly the main weaknesses of the Pareto criterion.

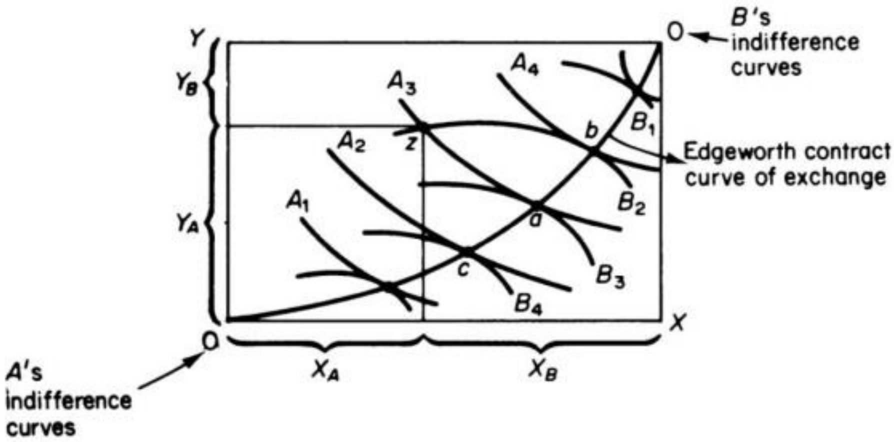
The Pareto criterion cannot evaluate a change that makes some individuals better- off and others worse-off. Since most government policies involve changes that benefit some and harm others it is obvious that the strict Pareto criterion is of limited applicability in real-world situations.

Furthermore, a Pareto-optimal situation does not guarantee the maximisation of the social welfare. For example, we know that any point on the production possibility curve represents a Pareto-efficient situation. To decide which of these points yields maximum social welfare we need an interpersonal comparison of the individual consumer's utility. In a subsequent section we will show that the Pareto- optimal state is a necessary but not sufficient condition for maximum social welfare.

Let us examine now the three marginal conditions that must be satisfied in order to attain a Pareto-efficient situation in the economy.

### (a) Efficiency of distribution of commodities among consumers

Applying the Pareto optimality criterion to the case of distribution of commodities Y and X, we can say that a distribution of the given commodities X and Y between the two consumers is efficient if it is impossible by a redistribution of these goods to increase the utility of one individual without reducing the utility of the other. In the above figure, we show the Edgeworth box for the given commodities X and Y. We know that only points on the Edgeworth contract curve satisfy the Pareto-optimality condition. Any other distribution off the contract curve is inefficient. For example, point z is inefficient, since a redistribution of the commodities such as to reach any point between a and b increases the utility of both consumers. A movement to a increases the utility of B without reducing the utility of A. Similarly, the distribution implied by b increases the utility of A without reducing the utility of B. Thus all the points from a to b represent improvements in social welfare compared with the distribution at z. By reversing the argument it can be seen that a movement from a point on the contract curve to a point off it results in a decrease in social welfare. Thus the contract curve shows the locus of Pareto-optimal or -efficient distribution of goods between consumers. This curve is formed from the points of tangency of the two consumers' indifference curves, that is, points where the slopes of the indifference curves are equal In other words, at each point of the contract curve the following condition is satisfied




MRs:.y = MRs:.y

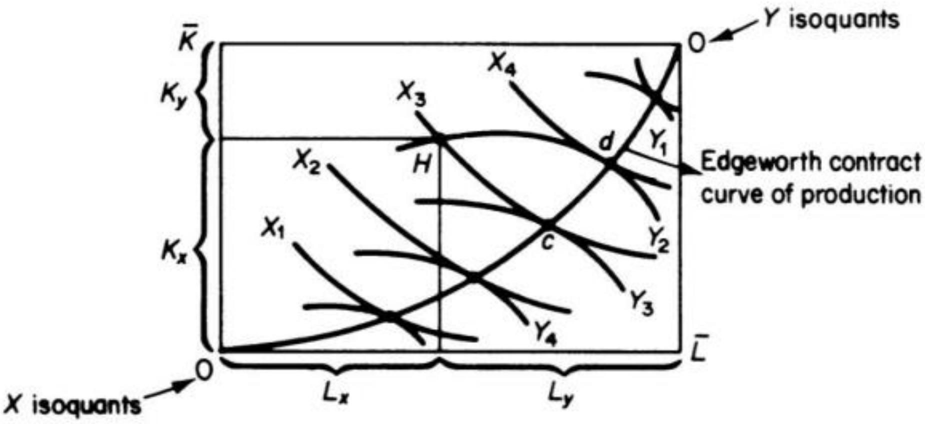
Therefore we may state the marginal condition for a Pareto-efficient distribution of given commodities as follows:

*The marginal condition for a Pareto-optimal or -efficient distribution of commodities among consumers requires that the MRS between two goods be equal for all consumers.*

**(b) Efficiency of allocation of factors among firm-producers**

To derive the marginal condition for a Pareto-optimal allocation of factors among producers we use an argument closely analogous to the one used for the derivation of the marginal condition for optimal distribution of commodities among consumers. In the case of allocation of given resources K and L we use the Edgeworth box of production shown in figure 2.

Only points on the contract curve of production are Pareto-efficient. Point H is inefficient, since a reallocation of the given K and L between the producers of X and Y such as to reach any point from c to d inclusive results in the increase of at least one commodity without a reduction in the other. The



contract curve is the locus of points of tangency of the isoquants of the two firms which produce X and Y, that is, points where the slopes of the isoquants are equal. Thus at each point of the contract curve the following condition holds

**$MRTS^{X}_{LK} = MRTS^{Y}_{LK}$**

Therefore we may state the marginal condition for a Pareto-optimal allocation of factors among firms as follows:

*The marginal condition for a Pareto-optimal allocation of-factors (inputs) requires that the MRTS between labour and capital be equal for all commodities produced by different firms.*

**(c) Efficiency in the composition of output (product-mix)**

The third possible way of increasing social welfare is a change in the product-mix. To define the third marginal condition recall that the slope of the PPC is called the 'marginal rate of (product) transformation' (MRPTxy), and it shows the amount of Y that must be sacrificed in order to obtain an additional unit of X. In other words the MRPT is the rate at which a good can be transformed into another.

The marginal condition for a Pareto-optimal or -efficient composition of output requires that the MRPT between any two commodities be equal to the MRS between the same two goods:

$MRPT_{x,y} = MRS^{A}_{XY} = MRS^{B}_{XY}$

Since the MRPT shows the rate at which a good can be transformed into another (on the production side), and the MRS shows the rate at which consumers are willing to exchange a good for another, the rates must be equal for a Pareto-optimal situation to be attained. Suppose that these rates are unequal. For example assume

That, $MRPT_{x,y} = 2Y/1X$ and $MRS_{x,y} = 1Y/1X$

$MRPT_{x,y} > MRS_{x,y}$

The above inequality shows that the economy can produce two units of Y by sacrificing one unit of X, while the consumers are willing to exchange one unit of Y for one unit of X. Clearly the economy produces too much of X and too little of Y relatively to the tastes of consumers. Welfare therefore can be increased by increasing the production of Y and decreasing the production of X.

*In summary. A Pareto-optimal state in the economy can be attained if the follow- ing three marginal conditions are fulfilled:*

**1. The $MRS_{x,y}$ between any two goods be equal for all consumers.**

**2. The $MRTS_{LK}$ between any two inputs be equal in the production of all commodities.**

**3. The $MRPT_{x,y}$ be equal to the $MRS_{x,y}$ for any two goods.**

# The Kaldor-Hicks Compensation Principle

Nicholas Kaldor and J.R.Hicks put forwarded the welfare criterion based upon the compensating payments in 1939. *If a certain change in economic organization or policy, according to Kaldor, makes some people better off and the others worse off, there will be a net increase in social welfare, when the gainers in welfare compensate the losers and are still better off than before.*

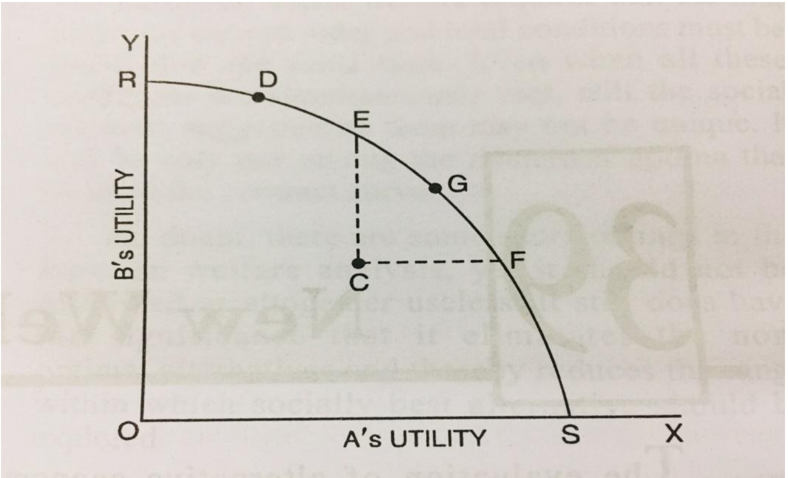
Assumptions:-

1. An individual himself is the best judge of his satisfaction which is independent of the satisfaction of others.
2. There is constancy of the tastes of the individuals.
3. There can be ordinal measurement of utility.
4. The inter-personal comparisons of utilities are not possible.
5. There is an absence of externalities in production and consumption.

Suppose there are two economic situation A and B. The former is socially preferable to B, in case those who gain from A can compensate the losers i.e. they can bribe the losers to accept the situation A and can still be better off than in situation B. For instance, the chemical factory in an area causes suffering to the people in its vicinity but the factory management compensates them and still the total product obtained by the society is more than what it would have been, had the factory been closed, there is a net increase in the welfare of the society.

J.R.hicks put forwarded his compensation criterion which is the reverse of Kaldor's criterion. According to him, if an economic change makes some people better off and the others worse off and the losers cannot bribe the gainers not to make the given change, then there is an increase in the net welfare of the society from that economic change. Suppose the government proposes to impose a tax which will benefit some and harm others. If the latter fail to bribe the government officials not to introduce the proposed tax, then the given change in tax policy will result in a net increase in the welfare of the society. *The compensation principle suggested by Kaldor and Hicks, even though stated in different words, actually means the same thing. The only difference is that Hicks has given his criterion from viewpoint of losers, whereas kaldor formulated his criterion from viewpoint of gainers.*

In the given figure, A's utility is measured along the horizontal scale and B's utility along the vertical scale. RS is the utility possibility curve. It indicates the different combinations of the utilities obtained by A and B. The movement downwards along the curve RS shows an increase in the utility of A and a




fall in the utility of B. On the opposite, an upward movement along RS curve shows a fall in the utility of A and a rise in utility of B. these movements along the utility possibility curve may be on account of redistribution of income.

Suppose the two individuals A and B are initially at the point C inside the utility possibility curve. If some change takes place in the economic policy and A and B moves to position D from C, there is a fall in the utility of A whereas there is an increase in the utility of B. It is not possible to evaluate the movement from C to D from Paretian criterion. The points E, F and G lying upon EF segment of the utility possibility curve RS are socially preferable to the point C on the basis of Paretian criterion as at least one individual becomes better off while none is worse off at the points E,F and G. As there is movement from C to D, it has to be seen if the individual B (the gainer) can compensate the individual A (the loser) and still be better off than C.

*According to Kaldor-Hicks criterion, as B compensates A, the income redistribution causes the movement from D to E where A's utility remains as before but B is still better off. It signifies that there is a net increase in the welfare of the society just by the redistribution of income.* Even the movement from D to G will result in a net increase in social welfare because at G both A and B are better off than at the original position C. According to Kaldor-Hicks criterion, it is not necessary to pay compensation actually to judge whether the social welfare has increased or not. Only requirement is that the gainer should be potentially able to compensate the loser for the loss in his welfare and still be better off.

Criticism:-

1. This criterion implicitly assumes that marginal utility of money is the same for all the individuals in the society.
2. It suggested potential compensation rather than actual compensation. In case the potential compensation is not paid, the welfare would be measured only in terms of utility and that would require the inter personal comparisons of utility and value judgements.
3. It evaluates the gains and losses due to an economic change in money terms. The real value of gains and losses have been overlooked.

It ignores the existing distribution of income of the community. If the income distribution is unequal, the compensation by the gainers (the rich) paid out to losers (the poor) will fail to offset the loss on account of the differences in the marginal utility of money in case of the rich and the poor.

CAREER STUDY
The Learning App

**SUPPORT US:**

**PLAY STORE** – CAREER STUDY

**YOUTUBE** – CAREER STUDY

**FACEBOOK** – CAREER STUDY

**WHATSAPP** – CAREER STUDY

**LEARN. GROW. SUCCEED.**