PHPSEC501

LOGICAL REASONING II

Unit I

Primacy of logical reasoning
Anumana : Definition, Constitution,
Processes and types.
Paksata, Paramarsa and Vyapti

1. What is Logical reasoning?

Ans: Logical reasoning is a form of thinking in which premises and relations between premises are used in a rigorous manner to infer conclusions that are entailed (or implied) by the premises and the relations. Different forms of logical reasoning are recognized in philosophy of science and artificial intelligence. *Deductive reasoning*, considered typical of mathematics, starts with premises and relations, which lead to a conclusion. For example, if $A = B$ and $B = C$ (the premises), the inevitable conclusion is that $A = C$ because equality is a transitive relation. Note that if $A \neq B$ and $B \neq C$, it is not possible to draw the conclusion that $A \neq C$ because inequality is not a transitive relation. *Inductive reasoning*, necessary in empirical sciences, uses observations to arrive at premises as well as relations between premises, which are then used to arrive at conclusions. Inductive reasoning cannot convey the certainty of deductive reasoning.

2. What is the primacy of logical reasoning?

Ans: Logic has frequently been defined as the science of reasoning. That definition, although it gives a clue to the nature of logic, is not quite accurate reasoning is that special kind of thinking called inferring, in which conclusions are drawn from name is this. As thinking, however, reasoning is not the special province of logic, but part of the psychologists subject matter as well. Psychologists who examine the reasoning process find it to be extremely complex and highly emotional, Consisting of awkward trial and error procedures that are illuminated by sudden and sometimes apparently irrelevant flashes of insight. These are all of importance to psychology. Logicians, however, are not interested in the actual process of reasoning, but rather with the correctness of the completed reasoning process. There question is always does the conclusion that is reached follow from the premises used or assumed? If the premises provide adequate grounds for accepting the conclusion, if asserting the premises to be true warrants asserting the conclusion to be true also, then the reasoning is correct. Otherwise, the reasoning is incorrect. Logicians methods and techniques have been developed primarily for the purpose of making this distinction clear patients are interested in all reasoning, regardless of its subject matter, but only from this special point of view.

3. Define Anumana. What is the structure of Anumana?

Ans: Anumana takes the form of syllogism. Anu means after and mana means knowledge which comes after some other knowledge. Anumana uses knowledge derived from perception or verbal testimony and arrives at something not given by perception. Anumana or inference as defined thus, is a process of reasoning in which we pass from the perception or some mark to the knowledge of something else by virtue of the relation of universal concomitance between the two for example

The hill is fiery.

Because it smokes.

And whatever smokes is fiery.

In this example we pass from the perception of the mark smoke in the hill to the knowledge of the existence of fire in it by virtue of our previous knowledge of the universal concomitance between smoke and fire.

In every anumana there are three terms namely sadhya, paksha, hetu. These 3 terms correspond to the major term, minor term and middle term respectively of the Greek syllogism. Hetu is also called linger.

Anumana is that kind of process of reasoning which is preceded by the perception of hetu in the paksha and the remembrance of its invariable concomitance with the sadhya. So in the process of Anumana-

1. The first step is the apprehension of the hetu in the paksha

2. The second step is the Recollection of the universal concomitance between the relation of hetu and sadhya.

3.The last step is the inferential knowledge of sadhya as related to in the paksha.

From these it appears that in anumana there must be at least three propositions which are all categorical and one must be affirmative and the others may be affirmative or negative. The first proposition corresponds to the conclusion. The second proposition corresponds to the minor premise and the 3rd to the major premise.

Thus we find that the order of arrangements and the propositions of anumana is the reverse of the arrangements of western syllogism.

4. What is vyapti? How is vyapti known or established?

Ans: The relation of invariable and unconditional coexistence between the middle term and the major term of an inference is technically called vyapti in Indian philosophy. For example,

Devadatta is mortal.

Devadatta is a man.

Therefore all men are mortal.

In these inference man is the middle and mortal is the major term. And the relation of invariable concomitance or coexistence between these two term is vyapti. Vyapti literally means the state of pervasion. It implies a coexistence between 2 facts someone named lee we appear which is pervaded and we are packer which is pervades.

Vyapti may be of two kinds, namely sama vyapti and ashama vyapti. A vyapti between 2 terms of equal extension is called sama vyapti. Here the vyapti holds between 2 terms which are coextensive so we may infer one of them from the other. For example, whatever is nameable is knowable and also whatever is knowable is nameable. A vyapti between two term of unequal extension, on the other hand, is called asama vyapti. The vyapti holds between 2 terms which are not coextensive. So from one we may infer the other but not vice versa. For example wherever there is smoke there is fire but whenever there is fire there may not be smoke as in the case of red hot iron ball. Now the question is how is vyapti known or how do we get a universal proposition.

According to Carvaka there is no such problem because to them inference is not a valid source of knowledge. The Buddhists base the knowledge of universal proposition on the principles of causality and essential identity. The vedantins hold that the knowledge of vyapti is the result of an unscientific induction.

The Naiyayikas agree with the vedantins but unlike them they supplement uncontradicted experience by few more step. So, according to nyaya, vyapti can be established on the basis of observation of particular instances through the knowledge of the class essence underlying such particular instances.

5. What are the different classifications of Anumana?

Ans: The Naiyayikas gives us the different classification of anumana according to their different principles according to the purpose, which an inference serves, anumana is of two kinds, namely,

1. Swarth anumana is that in which we infer to acquire knowledge for our own selves. If we infer the existence of fire in the hill and perceiving smoke in the hill remembering the universal relation between the smoke and the fire, then it is called swathanumana.

2. Parathanumana, on the other hand is that in which we infer to demonstrate the truth of the acquired knowledge before others. It is a syllogistic expression which consists of five members. They are-

A . pratijna - the heel is fiery.

B. Hetu - for it has a smoke.

C. Udaharana - wherever there is smoke there is fire as in the kitchen.

D. Anayaya - the heel has smoke.

E. Nigamana - the hill is fiery.

According to the causal relation, anumana is of three kinds, namely

1. Purvavat Anumana is that in which we infer the unperceived effect from a perceived cost. For example, when we infer future rain from the perception of dark cloud in the sky our inference is purvavat.

Two. Seshavat anumana is that in which we infer the unperceived Causeway from a perceived effect. For example, when we infer past rain from the perception of swift muddy current of the river, our inference is seshavat.

3. Samanyatadrsta anumana is that which is based on certain observed points of general similarity between the objects of experience. For example when a man infers cloven hoof of an unknown animal simply by seeing it's horn, we have an example of samanyatadrsta anumana.

According to the application of the method anumana is of three kinds, namely

1. Kavalanvayi anumana is that in which the middle term is positively related to the major term. This means that there is agreement between the two term in respect of prisons don't stop for example,

All smoky objects are fiery.

The heel is smokey.

Therefore the heel is fiery.

Here, the middle term Smokey is positively related to the major term fiery.

2. Kevalavyatireki anumana is that in which the middle term is negatively related to the major term this means that there is agreement between the two terms in respect of absence. For example,

No non fiery object Is Smokey.

The hill is smoky.

Therefore the hill is fiery.

Here, the middle term Smokey is negatively related with the major term non fiery.

3. Anvayi-vyatireki anumana is that in which the middle term is related to the major term both positively and negatively. For example

1. All smoky objects are fiery.

The hill is smoky.

Therefore the hill is fiery.

2. No non fiery object is smokey.

The heel is smokey.

Therefore the heel is fiery.

In this example there is double agreement between middle and major term. They agree in presence in the positive instance and they also agree in absence in the negative instance.

6. What do you mean by paksata?

Ans: In Sanskrit dictionary, Pakshata means 1) Alliance, partisanship. 2) Adherence to a party.

7. What is the paksha in Indian logic?

Ans: According to the nalysis in Indian logic (paksha, "the hill") "possesses" the middle term (hetu, "smoke"), which is recognized as "pervaded by" the major (sadhya, "fire"). The relation of invariable connection, or "pervasion," between the middle (smoke) and the major (fire)—"Wherever there is smoke, there is fire"—is called vyapti.

8. What is a Paksa in Nyaya philosophy?

Ans: Paksa is one of the three terms in constituents of Inference in Anumana (Inference- Nyaya). The word indicates after knowledge and it follows perception. It is defined as a specific form of knowledge preceded by perception.

9. What is Linga Paramarsha?

Ans: Paramarsha or Linga-Paramarsha or reference. Inference begins from observing the presence of proof or Hetu in the sadhya or Pakshya where it has been intimated by invariable correlation or Vyapti with the paksa or Sadhya.

10. Write a note on paramarsa.

Ans: Paramarsa can be paraphrased as 'the confirmatory knowledge'. It is a stage in the process of Inference. Inference is one of the four processes of knowing or internalizing the universe, accepted by the Indian logicians Naiyayikas). The other three processes are perceptual, analogical and verbal processes.

Although other systems of Indian philosophy accepted different numbers of the process of internalization, the Naiyayikas included all extra processes in one or the other of these four Pramanas. Carvaka accepted only one namely, perception. The Vaisesikas and the Buddhists accepted only two namely perception and Inference. The Sankhya philosophers accepted three adding verbal testimony to the above List of two. Naiyayikas accepted four by adding upamana to the above list. The Prabhakara school of Purva- mimamsa added presumption making the number five. The Bhatta school of Purva—mimamsa made it six by adding the process of knowing absence. The folk tradition added two more to the list namely history and inclusion.

The Naiyayikas included arthapatti and sambhava in inference aitihya in sabda and anupalabdhi was felt to be unnecessary since absence could be known by perception also. By 10th century A.D. the Vaisesikas and the Naiyayikas came very close in their world view and the system was more or less known as Nyaya-Vaisesika system of though. Jayanta Bhatta has clearly stated that Vaisesikastu asmad-anuyayinah eva. (Nyayamanjari Ist ahnika). The Nyaya-Vaisesika system of thought accepted five member process of anumana to convince others. The bhatta school accepted only three members. The five members are:

1. Pratijna : The Mountain possesses fire
2. Hetu : Because there is smoke
3. Udaharana : Like the Kitchen
4. Upanaya: Such is this mountain
5. Nigamana : Therefore Mountain Process fire

The Bhatta school thinks either the first three steps or the last three steps are sufficient to explain the generation of inferential cognition. This issue is being debated for more than 1500 years. Sasadhara the pre-gangesa neo-logician wrote an independent essay on Paramarsa in his Nyayasiddhanta dipa and defended the Nyaya-Vaisesika position taking into account all the pros and cons of the objection raised by the Bhatta-Mimamasakas.

Unit II

Hetvabhasa: Definition and types
Ashudha, Badhita, Satpratioaksa, Virudha, Svabhyabhichara

1. What is hetvabhasa?

Ans: The fallacies of inference (hetvābhāsa) in Indian logic are all material fallacies. So far as the logical form of inference is concerned, it is the same for all inferences. There Is, strictly speaking, no fallacious form of inference, in logic since all inferences must be put in one or other of the valid forms. Hence if there is any fallacy of inference, that must be due to the material conditions on which the truth of the constituent premises depends. It may be observed here that in the Aristotelian classification of fallacies into those in dictione and those extra dictionem there is no mention of the formal fallacies of inference like the undistributed middle, the illicit process of the major or minor term, and so forth. The reason for this, as Faton rightly points out, is that "to one trained in the arts of syllogistic reasoning, they are not sufficiently persuasive to find a place even among sham arguments." As for Aristotle's fallacies in dictione, i.e., those that occur through the ambiguous use of words, they are all included by the Naiyayika among the fallacies of chala, jāti and nigrahasthāna with their numerous subdivisions.

In Indian Logic, a material fallacy is technically called hetvābhāsa, a word which literally means a hetu or reason which appears as, but really is not, a valid reason. The material fallacies being ultimately due to such fallacious reasons, the Naiyāyikas consider all these as being cases of hetvābhāsa. According to the Naiyayi- 5 kas, there are five kinds of material fallacies. Taese are (1) Savyabhicara Satpratipaksa, (4) Asiddha, (5) Badhita.

2. Mention the different types of fallacies in Indian logic according to Nyaya.

Ans: The first kind of fallacy is called savyabhicara or the irre- gular middle. To illustrate:

All bipeds are rational;

Swans are bipeds

Therefore swans are rational.

The conclusion of this inference is false. But why? Because the middle term 'biped' is not uniformly related to the major 'rational' It is related to both rational and non-rational creatures. Such a middle term is called savyabhicara or the irregular middle.

The savyabhicara hetu or the irregular middle is found to lead to no one single conclusion, but to different opposite couclu- sions. This fallacy occurs when the ostensible middle term violates the general rule of inference, namely, that it must be universally related to the major term, or that the major term must be present in all cases in which the middle is present. The savyabhicara middle, however, is not uniformly concomitant with the major term. It is related to both the existence and the non-existence of the major term, and is, therefore, also called anaikāntika or an inconstant concomitant of the major term. Hence from such a middle term we can infer both the existence and the non-existence of the major term. To take another illustration:

All knowable objects are fiery;

The hill is knowable;

Therefore the hill is fiery;

Here the middle 'knowable' is indifferently related to both fiery objects like the kitchen, and fireless objects like the lake. All knowables being thus not fiery, we cannot argue that a hill is fiery because it is knowable. Rather, it is as much true to say that, for the same reason, the hill is fireless. " The second kind of fallacy is called viruddha or the contra dictory middle. Take this inference: "Air - is heavy, because it is empty." In this inference the middle term 'empty." is con- tradictory because it disproves the heavi ness of air. Thus the viruddha or the contradictory middle is one which disproves the very proposition which it is meant to prove. This happens when the ostensible middle term, instead of proving the existence of the major, in the minor, which is intended by it, proves its non-existence therein. Thus to take the Naiyayikas' illustration, if one argues. "Sound is eternal, because it is caused," we have a fallacy of the viruddha or contradictory middle. The middle term, 'caused does not prove the eternality of sound, but its non-eternality, because whatever is caused is non-eternal. The distinction between the savyabhicāra and the viruddha is that while the former only fails to prove the conclusion, the latter disproves it or proves the contradictory proposition.

The third kind of fallacy is called satpratipaksa or the infer entially contradicted middle. This fallacy arises when the ostensible middle term of an inference is validly contradicted by other middle term which proves the non-existence of the major term of the first inference. Thus the inference "sound is eternal, because it is audible" is validly contradicted by another inference like this: "sound is non-eternal, because it is produced like a pot. Here the non-existence of eternality (which is the major term of the first inference) is proved by the second inference with its middle term 'produced' as against the first inference with its middle 'audible.' The distinction between the viruddha and the satpratipakasa is that, while in the former the middle itself proves the contradictory of its conclusion, in the latter the con tradictory of the conclusion is proved by another inference based on another middle term.

The fourth kind of fallacy is called asiddha or sādhyasama, i. e. the unproved middle. The sadhya-sama middle is one which is not yet proved, but requires to be proved, like the that the sadhyasama middle is not a proved or an established fact, but an asiddha or unproved assumption. The fallacy of the asiddha occurs when the middle term is wrongly assumed in any of the premises, and so cannot be taken to prove the truth of the conclusion. Thus when one argues, "the sky-lotuss is fragrant because it has lotusness in it like a natural lotus," the middle has no locus standi, since the sky-lotus is non- existent, and is, therefore, asiddha or a merely assumed but not proved fact.

The last kind of fallacy is called bädhita or the non-infer entially contradicted middle. It is the ostensible middle term of an inference, the non-existence of whose major is ascer- tained by means of some other promāna or source of knowledge. This is illustrated - "Fire is cold because it is a substance". Here 'coldness' is the sadhya or major term, and 'substance' is the middle term. Now the non-existence of coldness, may more, the existence of hotness is perceived in fire by our sense of touch. So we are to reject the middle 'substance' as a con- tradicted middle. The fallacy of satpratipaksa, as explained before, is different from this fallacy of badhita, because in the former one inference is contradicted by another inference, while in the latter an infe ence is contradicted by perception or some other non-inferemial source of knowledge. Another example of badhita would be: Sugar is sour, because it produces acidity.

Unit III

Symbolisation, Construction of truth table for Statement forms
Truth Table Technique for testing validity for invalidity of arguments

1. Symbolic logic is entirely formal in nature- is it true?

Answer:- Yes

2. Not is/ is not a logical constant.

Answer:- Is

3. Application of ideogram is /is not a characteristic of symbolic logic.

Answer- Is

4. Symbolic logic uses deductive/inductive method.

Answer:- Deductive.

5. If p is true and q is also true, what is the value of

$\sim p \lor q$?

Answer- true.

6. 'v' represents conjunction/ disjunction.

Answer:- Disjunction.

7. Symbolize – 'If I have money and also friends then I can organize the function'.

Answer. $(p.q) \supset r$

8. Variable / is not a logical constant.

Answer :– is not

9. p v q stands for either he will not attend the meeting or he will - Is it true?

Answer- no

10. Phonogram stands for sound.

11. Ideogram stands for sound/concept.

Answer:- concept.

12. A variable is/ is not a symbol

Answer:- Is.

13. Illustrate variable.

Answer- p,q,r,s.

14. Symbolize 'he is neither intelligent nor industrious'.

Answer- $\sim p \lor \sim q$

15. Find the value of $\sim (\sim q \supset \sim p)$, if truth is the value of both the variables.

Answer :- $\sim(\sim q \supset \sim p)$

$= \sim (\sim T \supset \sim T)$

$= \sim (F \supset F)$

$= \sim T$

$= F$

16. What is symbolic logic?

Answer:- Symbolic logic is the name given to that part and the method of treatment of formal logic which is comparatively of recent origin and in which specialized symbols are used extensively to express the arguments as to determine their value in a better and easier way.

17. What is symbol?

Answer:- Symbol is a sign which is consciously designed to stand for something, for example, the national flag of a nation is the symbol of that nation.

18. Define variable.

Answer:- Generally variable means changeable. In logic variable means a symbol which can stand for anyone of the given range of values. Variables are those symbols which do not have any fixed meaning. They can be given any range of values.

19. Define constant.

Answer: Logical constants are those symbols which exhibits the form of compound proposition and which bears the same meaning in the symbolic expression of proposition the following are the logical constants.

Negative (not) – '~' (curl)

And- '.' (dot)

If… then – '⊃' (horseshoe)

Either… or – 'v' (vel)

Equivalence (if and only) – ' ≡' (three bar)

20. Define classical logic.

Answer:- Classical logic is the name given to those parts and that method of treatment of formal logic which have come down substantially are changed from classical and medieval time.

21. What is the relation between symbolic and classical logic?

Answer- 1. Both the two systems are concerned with forms and validity of arguments.

2. Modern logic is the development of concepts and technique which were implied in the work of Aristotle, the founder of classical logic.

22. What is called a truth value of a proposition?

Answer:- Truth or falsity is the very essence or character of a proposition. The truth or falsity of a proposition is called its truth value.

23. What is to table method?

Answer- The method or technique in which the truth table are used to determine the validity or invalidity of an argument is called the truth table method.

24. What are the two different types of symbol?

Answer- Symbols are of two types, namely verbal and nonverbal. Different words used to indicate substance, quality, action etc are called verbal symbols, for example books, honesty,come, etc. Different letters such as p, q, r, s etc or mathematical symbols such as +, -, ÷, × etc are nonverbal symbols. The nonverbal symbols are used in symbolic logic.

25. Distinguish between sign and symbol.

Answer- Symbol & sign differ. It is not easy to draw a hard and fast line between the two. Sign may be natural or artificial but symbols are always artificial. Smoke is the sign of fire because there is natural and necessary relation between smoke and fire but symbol is that sign which is used consciously to indicate something. The national flag is a symbol of a nation. It is consciously designed. Symbols are predetermined artificial sign which is not always in the case of sign.

26. Distinguish between phonogram and ideogram.

Answer: Ideogram are those symbols which stands directly for concepts, for example multiplication sign 'X' and such other signs are ideograms. Ideograms are often contrasted with phonograms. Phonograms are signs which stands directly for sounds and indirectly for concepts, for example the written word 'multiplication sign' is phonogram. Unlike the ideograms the phonogram represents spoken English words which correspond to them while phonograms refer to the words in language which are written according to some sorts of phonetic rules, ideograms refers to the concepts or the ideas represented by the words.

27. What is truth function?

Answer:- A truth function is a compound expression where the value of the expression is determined by the value of the constituent variables the value of the statement or compound expression is determined as soon as the truth value of the constituent variables are known, for example the statement form p v q is a truth function because the truth value of the expression is determined by the truth values of the two constituent variables p and q.

28. What is it truth table?

Answer: Truth table is a convenient way to show the truth values of a truth function taking into consideration all possible truth values of a propositional variables involved in it. So the truth table is a definition of the truth function the compound sentence 'p and q' is called a truth function of its component sentences p and q because when we know the truth value of the compound sentence is thereby determined the tabular way of representing the dependence is called a truth table.

29. What is proposition?

Answer:- A proposition is a sentence or combination of sentence which is either true or false, for example Ram is a boy, 5 + 2 = 7 etc. Similarly, Ram is not a boy, 5 + 2 = 10 etc are propositions because the properties of truth and falsity can be meaningfully applied to such statements. On the other hand, a property of truth and falsity cannot be applied to the statement do you know him?, this statement is therefore is not a proposition.

30. What are the two kinds of proposition?

Answer:- Propositions may be divided into two kinds, simple and compound. A simple proposition is one which consists of a single statement, for example you study hard, this can be symbolized as 'p'. A compound proposition is one which contain two or more statements as component, for example , Ram is a boy and Sita is a girl this is symbolically expressed as 'p and q'.

The following examples show the way in which various statements are joined and negated to form a compound propositions-

1. If you read then you will pass.

2. Either he will come or she will come.

3. You worked hard and you passed.

4. It is false that you worked hard.

Now the meaning of the above propositions is ignored the symbol p and q are used for the first and second statement then the proposition can be symbolically expressed in the following way-

1. If p then q

2. Either p or q

3. p and q

4. not p

31. What do you mean by tautology or tautologous proposition?

Answer- The tautologous proposition is that kind of statement which is true under all conditions. The truth of such proposition is independent of any experience, for example he is a philosopher. The truth of the proposition can be known without any experience. The proposition is necessarily true. The statement form of tautologous proposition is ‘p v ~ p’. To show that the statement form is a tautology let us construct the following truth table-

p p v ~p

TT T F

FF T T

As there is only one variable so there are only two rows. In the conclusion column of this two table there is only T’s this fact shows that all of the substitution instances are true that means this statement form is tautology.

32. What do you mean by contradictory proposition or contradiction?

Answer: A contradictory proposition is that kind of statement which is false under all conditions. The falsity of such proposition is also independent of any experience, for example, he is and is not a philosopher. The falsity of the proposition can be known by itself independent of any statement form of contradictory proposition is p . ~ p. To show that the statement form is contradiction let us draw the following truth table-

p p . ~p

T T F F

F F F T

In the conclusion column of this truth table there is only F’s. This fact shows that all of its substitution instances are false that means this statement form is contradiction.

33. What do you mean by contingent proposition?

Answer:- A contingent proposition is that kind of statement which is true under such and such condition. The truth or falsity of this kind of proposition is verified on the basis of experience only. Contingent proposition is not necessarily true, for example Ram is a boy and Romi is a boy. The truth or falsity of this proposition is not self-evident, experience can only prove that the statement form of contingent proposition is ‘p . q’, ‘p v q’, p ⊃ q etc. To show that this statement form is contingent, let us draw the to table of the following

p q p.q

T T T

T F F

F T F

F F F

As there are two variables so there are four rows. In the conclusion column there are T&F both. This fact shows that this statement form is contingent as its truth value is dependent on its content rather than its form alone.

34. What is the nature of symbolic logic?

Answer- 1. Symbolic logic is a branch of logic. It is of recent origin. It is only about 180 years old as the first statement of symbolic logic as an independent branch appears in the books written by Augustus de Morgan and George Boole.

2. Symbolic logic is also concerned with the forms and validity of arguments like that of classical logic.

3. Symbolic logic uses symbols extensively because according to symbolic logicians, by use of special symbols validity or invalidity of arguments can be easily and clearly determined.

35. Distinguish between classical logic and symbolic logic.

Answer – 1. In classical logic symbols were used to a limited extent whereas in symbolic logic symbols are used extensively.

2. Modern logic clearly draws a distinction between inference and implication, in traditional logic this distinction was not clearly made.

3. Regarding the analysis of proposition in classical logic every categorical proposition is regarded to be the statement of certain relation between two terms. In modern logic, a proposition need not be two termed proposition.

4. The classical logicians thought that the capula alone is capable of expressing all types of relations. Modern logicians recognize various types of relations such as symmetry, transitivity etc.

5. Again regarding the classification of proposition the classical logic classify proposition into two types, namely categorical and conditional. Modern logicians in contrast, classified propositions into simple, compound and general. Further, on the basis of the idea of truth functionality, propositions are classified into tautologous, contingent and contradictory.

36. What are the characteristics of symbolic logic?

Answer- According to a distinguished American philosopher, C. I. Lewis the basic characteristics of symbolic logic are as follows:-

1. The use of symbols: The first basic characteristics of symbolic logic is the use of special symbol, for example '?' is ideogram but the written word 'question mark' is phonogram. It directly represents the spoken English which corresponds to it and not concept. According to symbolic logicians, the symbols have certain advantages, such as symbols bring out clearly the forms of arguments for which it become easier to divide arguments into valid and invalid.

2. The use of deductive method: The second basic characteristics of symbolic logic is the use of deductive method. In the deductive method from a limited number of statement we can produce an infinite number of other statements. Symbolic logic implies this deductive method and in this respect symbolic logic is quite close to mathematics.

3. The use of variables : The third basic characteristic of symbolic logic is the use of variables. Generally variable means changeable in logic variable is a symbol which can stand for anyone of the given range of value.

37. Write a note on importance of using symbols.

Answer- 1. The use of symbols exhibits the logical form of the argument and thereby helps to classify them according to their forms so that their validity can be determined through the application of general rules.

2. The use of symbols help to express the generality of the logical rules.

3. The use of symbols give conciseness, clarity and economy of expression to complicated statements and arguments which would be practically unintelligible when expressed in ordinary language.

4. The use of symbols help to express conveniently the concepts and technical terms which are peculiar to logic. Further symbols like variables are also useful to represent different logical operation.

5. The use of symbol helps to safeguard the argument from the influence of the material truth or falsity of the constituent proposition which create difficulties in the determination of its validity.

38. Why do you symbolize ordinary language?

Answer- An argument contain proposition as its premises and conclusion. Arguments as expressed in ordinary natural language are often difficult to appraise in their proper form on account of the vagueness and ambiguity of the words, the misleading idioms etc. With the view to resolve difficulties of ordinary language the logicians have introduced the artificial symbolic language.

Hence, in symbolic language one important task is the symbolization of propositions and argument stated in ordinary language. In order to symbolize proposition and argument as expressed in ordinary language it is necessary to have a clear knowledge about proposition. However, it is also necessary to discuss clearly about the symbols- how and what type of symbols ought to be used, how proposition and their mutual logical relationships can be expressed adequately with the help of such symbols.

39. What points are to be noted about logical constants?

Ans: 1. Every compound proposition is formed by means of logical constants from simple propositions. The basic elements of any compound proposition, however complex are simple propositions and the logical constant form the compound proposition from the simple ones.

2. Every logical constants with exceptions of ‘not’ (negation) is known as logical connective because such logical constants form the compound propositions by connecting simple or other compound propositions.

3. Each connective connects two and only two propositions.

4. ‘Not’ or ‘it is false that…’ (negation) differs from other logical constants. Although it is a constant and extremely important one. It is not accepted as connective by some logicians that it does not serve to connect two propositions. But it is like other constant that it serves to form compound propositions from simple propositions by negating the simple proposition. In this sense, some other logicians accepted it as a connective.

40. Explain the truth functions of conjunction negation, disjunction, implication and equivalence.

Answer- The truth table of a compound proposition depends upon the truth value of a simple proposition out of which it is composed. The compound proposition is called a truth function of its component simple proposition for, the word or by which the simple propositions are joined together to form a compound proposition is called connective the words like ‘and’, ‘not’, ‘either… or’, ‘if… then’, ‘if and only if’ are propositional connectives.

The first type of connective Is conjunction. We can form the conjunction of two statements by placing the word ‘and’ between them. The statement so combined are called conjuncts. We produce‘.’ (dot) as our symbol for conjunction thus ‘p’ & ‘q’ any two statements whatever their conjunction is written as ‘p.q’. The truth value of the conjunction of two statements is determined wholly by the truth value of its two conjuncts there are only four possible cases and the truth value of the conjunction in each are as follows which is represented by means of a truth table -

p q. p.q

T. T. T

T F F

F. T. F

F. F. F

The negation or contradictory of statement is often formed by inserting a ‘not’ into the original statement. For negation we use the symbol ‘~’ (curl). The negation of any true statement is false and the negation of any false statement is true now this fact can be represented by means of a truth table - –

p. ~p

T F

F T

A disjunctive truth function of proposition and compound proposition in which two propositions are joined together into one by the symbol 'v' (vel). The two component statements so combined are called disjuncts. The Latin word 'vel'signifies inclusive disjunction and we use the letter 'v'. The word 'vel'signifies 'or'in this sense where p and q are any statement whatever, their inclusive disjunction is written as 'p v q' . This can be represented by the following two table-

p. q. p v q

T. T. T

T. F. T

F. T. T

F. F. F

A compound proposition can be obtained by the help of connective 'if….. then'. A compound statement so obtained is called implicative function of the compound proposition. If p and q are two statements 'if p the q' is an implicative function of the truth table of p and q. For 'if…then' we use special symbol '⊃ ' (horseshoe) and thus 'if p then q' is symbolized as 'p⊃ q'. The implicative function of p ⊃ q is false when p is true and q is false. This can be represented by the following truth table –

p. q. p ⊃q

T. T. T

T. F. F

F. T. T

F. F. . T

An equivalent function is true only if p and q are either both true and both false otherwise it is false. p ≡q is an equivalent function. Here, there are two variables, so there will be four possible combinations of truth values its truth table may be regarded as the definition or formula of equivalent function . The tabular form of these-

p. q. p ≡q

T. T. T

T. F F

F. T. F

F. F.   F

41. How do you prove invalidity with the help of truth table?

Ans: Argument forms are important in modern logic. For the Rules provided by the modern logicians to test the validity of arguments are based primarily on the form of the argument and not on its content. As we have already said, the primary task of deductive logic is to distinguish valid arguments from invalid ones. We have already shown above that if the premises of a valid argument are true, then the conclusion must be true. We have also seen that if the conclusion of a valid argument is false, at least one of the premises must be false. In other words, the premises of a valid argument give incontrovertible proof of the conclusion drawn. Now we need to make this formal account of validity more precise. In order to do this we introduce the concept of argument form. Let us consider two examples, which evidently have the same form:

1. If Tagore wrote the plays attributed to Shakespeare, then Tagore is a great writer.

Tagore is a great writer.

Therefore Tagore wrote the plays attributed to Shakespeare.

Even if we agree with the premises of this hypothetical syllogism, we cannot agree with its conclusion. For, we can clearly see that this argument is invalid. One of the ways of proving its invalidity is to use the method of logical analogy. For we can as well argue:

2. If Nehru was assassinated, then Nehru is dead.

Nehru is dead.

Therefore Nehru was assassinated.

No one will seriously defend this argument because its premises are known to be true and the conclusion is known to be false. Therefore, this argument is obviously invalid. The form of this argument is the same as that of the first argument which is invalid. This way of refuting an argument is very effective. This way of refutation is known as refutation by logical analogy. This method points to an excellent technique of testing arguments. That is, to prove the invalidity of an argument it is enough to formulate an argument that (i) has exactly the same form as the first and (ii) has true premises and a false conclusion. This method is based on the fact that validity and invalidity are purely formal characteristics of arguments. In other words, any two arguments that have exactly the same form are either both valid or both invalid, no matter what the differences are in the subject matter with which they are concerned. A given argument becomes clear when the simple sentences in it are abbreviated by capital letters. We may thus abbreviate the statements, ‘Tagore wrote the plays attributed to Shakespeare,’ ‘Tagore was a great writer,’ ‘Nehru was assassinated,’ and ‘Nehru is dead,’ by the letters, T, G, A, and D respectively. In this manner we can easily symbolize the two sample

arguments given above as

$T \supset G$ $A \supset D$

G D

$\therefore T$ $\therefore A$

Comparison of these two examples illustrates what we mean by argument form. How do we obtain argument forms? We need a method to obtain them because we base our study on the forms of arguments only rather than on particular arguments having those forms. What applies to the form applies equally well to what conforms to such form. This is a sort of generalization very much akin to mathematical induction. This is made feasible by way of introducing the notion of variables. Modern logicians use, to avoid confusion, small or lowercase letters from the middle part of the alphabet, p, q, r, s, … as statement variables. A statement variable is simply a letter for which, we substitute a statement. Not only simple sentences but compound sentences also can be substituted for sentence (statement) variables. In the light of these considerations, we can now define what an argument form is: An argument form is any array of symbols which contains sentence variables (p, q, r, s, t) such that when sentences are substituted for the sentence variables – the same sentence replacing the same sentence variable throughout – the result is an argument. The form of two arguments is as follows:

$p \supset q$

q

$\therefore p$

This is an example for argument form. For, when the sentences T and G are substituted for the sentence variables p and q respectively, the result is the first argument given above. Similarly, if we substitute A and D for the sentence variables p and q, the result is the second argument given above. This leads us to the idea of what logicians mean by a substitution instance: Any argument that results from the substitution of statements for statement variables in an argument form is called a substitution instance of that argument form. Therefore if the form is invalid, then any argument which subscribes to this form is invalid. We have become familiar with one argument form. It corresponds to what traditional logic calls mixed hypothetical syllogism. Since we have already considered invalid argument form, it is desirable that we consider now a valid argument form.

3. If there is nuclear warfare, then humanity perishes.

There is nuclear warfare.

Therefore the humanity perishes.

It is obvious that this argument is valid. If this aspect is not clear, then we shall resort to logical analogy and consider another example.

4. If there is inflation, then the cost of living goes up.

There is inflation.

Therefore the cost of living will go up.

A cursory look at these arguments shows that they have identical structure. Therefore if one of them is valid, then the other one also must be valid. It is obvious that (4) is valid. It means that (3) also must be valid. Logical analogy is helpful only for a beginner who is not familiar with the irrelevance of matter of argument in a study of deductive logic. This is not the only argument form we come across. Traditional logic considered several other types of arguments; categorical syllogism, mixed disjunctive syllogism and pure hypothetical syllogism are the other types. All these types correspond to argument forms. Since in the last unit we had studied exhaustively several aspects of categorical syllogism, we can restrict ourselves to conditional arguments. Let us recall these arguments.

Mixed Disjunctive Syllogism:

a) p v q

~ p

∴ q

A disjunctive syllogism which conforms to this form is valid. Otherwise, it is invalid. So we shall consider the form of an invalid argument.

b) p v q c) p v q

p q

∴ q ∴ p

In our study of logic we must have a clear perception of what we have to do. We may have to examine several arguments and arguments with plurality of structures. In such a situation we have to reduce every argument to the form to which it corresponds. Proper identification or matching of argument and argument form is necessary. Any mismatch will lead us astray. Therefore this is an important step in our endeavour. This will also explain why we should be familiar with argument forms. Against this background, we shall examine examples. Though this is a repetition of what we studied earlier, considering the importance of argument form it is useful to do so.

5. Voters are either indifferent or ignorant.

Voters are not indifferent.

∴ Voters are ignorant.

6. Men are either humane or boorish.

Men are boorish.

∴Men are humane.

How should we match? A little care reveals that (a) matches 9 (it is unimportant whether first alternative is denied or second alternative is denied) whereas (b) and 10 match. Any argument is examined in this fashion only.

Consider the form of pure hypothetical syllogism.

a) $p \supset q$ b) $p \supset q$ c) $p \supset q$

$q \supset r$ $\sim p \supset r$ $p \supset r$

$\therefore p \supset r$ $\therefore \sim q \supset r$ $\therefore p \supset r$

We have three argument forms. Among them first two are valid whereas the last form is invalid. In fact except first two forms any other form is invalid. Let us construct arguments which match these forms.

7. If atmosphere is polluted, then life on this planet becomes extinct.

If life on this planet becomes extinct, then God does not exist.

∴ If atmosphere is polluted, then God does not exist.

8. If scientists are honest, then science will progress.

If scientists are not honest, then religion is strong.

∴If science does not progress, then religion is strong.

9. If politicians are patriots, then the country becomes prosperous.

If politicians are patriots, then democracy does not fail.

∴ If the country prospers, then democracy does not fail.

7 and 8 are valid. Therefore corresponding argument forms are also valid. 9 is invalid. Therefore corresponding argument form also is invalid. In other words, every argument has matter (content) and form. But it is the form that is fundamental from the point of view of validity. Modern logicians usually classify arguments according to the forms the arguments exhibit. Since it is possible, theoretically, to construct any number of arguments it is impossible to consider matter as the theme of deductive logic. Therefore form is the parameter to examine arguments.

As is clear from the above description, variables are symbols which can be replaced. There are three types of replaceable variables: class, individual, and sentential. Class variables are those replaceable by names of classes. For example, cats, horses, mangoes etc. are class variables.

Individual variables are those that are replaceable by the names of individuals. Newton, the tallest animal in the world, the most massive star, etc. are individual names. In other words, general terms signify class variables whereas singular terms signify individual variables. Later we will understand that the difference between these variables contributes to the difference in notations, a point which becomes clear when we undertake a study of quantification.

Sentential variables are those replaceable by the names of sentences. x is regarded as sentential variable and when a proposition consists of ‘x’, it is called propositional function. Propositional function is neither true nor false. It takes truth-value only when it is given some value. Value is given when the propositional variable is replaced by a proposition. All propositions are constants in contrast with propositional function. It is an accepted practice to represent constants with upper case letters while propositional function is always represented with lower case x. Similarly, we have already seen what is meant by a substitution instance. Let us clarify it a little more now. A substitution instance is any argument, which results from the substitution of sentences for the sentential variables of an argument form, is said to have that form, or to be a substitution instance of that form. For example, the argument ‘U v W and ~U; ∴W’ has the form

p v q and ~ p, ∴q

The simplest way of understanding argument forms is through truth-table. This is important for one more reason. Construction of truth-table is basic to our study of symbolic logic. We shall construct truth-tables to distinguish between valid and invalid forms.

Mixed hypothetical syllogism: p => q; p; □q

p q p ⊃ q p q

T T. T. T T

T F F. T F

F T T. F T

F F. T. F F

5th column stands for the conclusion. This table discloses that the conclusion is false only when one of the premises is false. In all other circumstances the conclusion is true. This is the condition of any valid inference. The same explanation holds good for the remaining argument forms.

Unit IV

Proof construction – Direct, Indirect and Conditional

1. What do you mean by formal proof of validity?

Answer- A formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise or that argument or follows from preceding statements by an elementary valid argument and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved. We first define an elementary valid argument as any argument is a substitution instance of an elementary valid argument form.

2. What is the formal way of writing out the proof or validity?

Answer- A more formal and more concise way of writing out this proof of validity is to list the premises and the statement deduced from them in one column with justification for the letter written beside them each case the justification for a statement specifies the preceding statements from which the rule of inference by which the statement in question was deduced this convenient to put the conclusion to the right of the last premises separated from it by a slanting line which automatically marks about it to be premises.

3. Illustrate an elementary valid argument.

Answer - Any substitution instance of an elementary valid argument form is an elementary valid argument thus, the argument

$\sim C \supset (D \supset E)$

$\sim C$

$\therefore D \supset E$

is an elementary valid argument because it is a substitution instance of the elementary valid argument form Modus Ponens (MP). It results from

$p \supset q$

p

$\therefore q$

But substituting '~C'for p & 'D ⊃ E'for q therefore it is of that form even though Modus Ponens is not the specific form of the given argument.

4. Why are the 9 rules of inference called elementary valid argument forms?

Answer- The nine rules of inference are elementary valid argument forms whose validity is easily established by two tables they can be used to construct formal proofs of validity for a wide range of more complicated arguments. The names listed are standard for the most part and the use of their abbreviations permit formal proofs to be set down with a minimum of writing.

5. Why do we use the formal truth of validity?

Answer- If argument contains more than two or three different simple statements or components, it is tedious to use through tables to test their validity. More convenient method of establishing the validity of some argument is to deduce their conclusion from their premises by a sequence of shorter, more elementary of arguments that are already known to be valid. Consider, for example, the following arguments form in which there are five simple statements

A v (B ⊃ D)

`C ⊃ (D ⊃ E)

A ⊃ C

~C

∴ B ⊃ E

To establish the validity of this argument by means of a truth table would require a table with 32 rows. We can prove the valid argument valid however by a sequence of just four arguments whose validity has clearly been established by using the rules of inference

1. A v (B ⊃ D)

2. ~C ⊃ (D ⊃ E)

3. A ⊃ C

4. ~C / ∴ B ⊃ E

5. ~A 3,4 MT

6. B ⊃ D 1,5 DS

7. D ⊃ E 2, 4 MP

8. B ⊃ E 6, 7 HS

Rules of inference-

1. Modus Ponens (MP)

p ⊃ q

p

∴ q

2. Modus Tolens (MT)

p ⊃ q

~ q

∴ ~p

3. Hypothetical syllogism (HS)

$p \supset q$

$q \supset r$

$\therefore p \supset r$

4. Disjunctive syllogism (DS)

$p \vee q$

$\sim p$

$\therefore q$

5. Constructive Dilemma (CD)

$(p \supset q) . (r \supset s)$

$p \vee r$

$\therefore q \vee s$

6. Absorption (ABS)

$P \supset q$

$\therefore p \supset (p.q)$

7. Simplification (simp)

$p.q$

$\therefore p$

8. Conjunction (conj)

p

q

$\therefore p.q$

9. Addition (Add)

p

$\therefore p \vee q$

Rule of Replacement: Any of the following logically equivalent expressions can replace each other wherever they occur:

10. De Morgan's Theorems (De M.):

$\sim(p.q) \equiv (\sim p \vee \sim q)$

$\sim(p \vee q) \equiv (\sim p. \sim q)$

11. Commutation (Com.):

$(p \vee q) \equiv (q \vee p)$

$(p.q) \equiv (q.p)$

12. Association (Assoc.):

$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$

$[p.(q.r)] \equiv [(p.q). r]$

13. Distribution (Dist.):

$[p.(q \vee r)] \equiv [(p.q) \vee (p.r)]$

$[p \vee (q.r)] \equiv [(p \vee q).(p \vee r)]$

14. Double Negation (D.N.):

$p \equiv \sim\sim p$

15. Transposition (Trans.):

$(p \supset q) \equiv (\sim q \supset \sim p)$

16. Material Implication (Impl.):

$(p \supset q) \equiv (\sim p \vee q)$

17. Material Equivalence (Equiv.):

$(p \equiv q) \equiv [(p \supset q).(q \supset p)]$

$(p \equiv q) \equiv [(p. q) \vee (\sim p. \sim q)]$

18. Exportation (Exp.):

$[(p.q) \supset r] \equiv [p \supset (q \supset r)]$

19. Tautology (Taut.):

$p \equiv (p \vee p)$

$p \equiv (p.p)$

Q. What is conditional proof?

Ans: To every argument there corresponds a conditional statement whose ante. cedent is the conjunction of the argument's premisses and whose consequent is the argument's conclusion. As has been remarked, an argument is valid if and only if its corresponding conditional is a tautology. If an argument has a conditional statement for its conclusion, which we may symbolize as AC then if we symbolize the conjunction of its premisses as P_{1} ^ 2 the argument is valid if and only if the conditional

$P \supset (A \supset C)$.   (1)

is a tautology. If we can deduce the conclusion $A \supset C$ by a sequence of elementary valid arguments, from the premisses conjoined in P, we thereby prove the argument to be valid and the associated conditional (1) to be a tautology. By the principle of Exportation, (1) is logically equivalent to

$(P.A) \supset C$.   (2)

But (2) is the conditional associated with a somewhat different argument. This second argument has, as its premisses, all of the premisses of the first argument plus an additional premiss that is the antecedent of the conclusion of the first argument. The conclusion of the second argument is the consequent of the conclusion of the first argument. Now, if we deduce the conclusion of the second argument, C, from the premisses conjoined in PA by a sequence of elementary valid arguments, we thereby prove that its associated conditional statement (2) is a tautology. But since (1) and (2) are logically equivalent, this fact proves that (1) is a tautology also, from which it follows that the original argument with one less premiss and the conditional conclusion A supset C -is valid also. Now, the rule of Conditional Proof permits us to infer the validity of any argument

P

$\therefore A \supset C$

from a formal proof of validity for the argument

P

A

$\therefore C$

Given any argument whose conclusion is a conditional statement, a proof of its validity, using the rule of Conditional Proof-that is, a conditional proof of its yalidity is constructed by assuming the antecedent of its conclusion as an additional premiss, and then deducing the consequent of its conclusion by a sequence of elementary valid arguments. Thus a conditional proof of validity for the argument

$(A \vee B) \supset (C.D)$

$(D \vee E) \supset F$

∴A ⊃ F

may be written as

1. (A v B) ⊃ (CD)

2. (DvE) ⊃ F /∴A ⊃ F

3. A/∴F (C.P.)

4. A v B 3, Add.

5. C.D 1, 4, M.P.

6. D.C 5, Com.

7. D 6, Simp.

8. DvE 7, Add.

9. F 2, 8, M.P.

Q. What is indirect proof?

Ans: The method of indirect proof, often called the method of proof by reductio ad absurdum, is familiar to all who have studied elementary geometry. In deriving his theorems, Euclid often begins by assuming the opposite of what he wants to prove. If that assumption leads to a contradiction, or 'reduces to an absurdity', then that assumption must be false, and so its negation-the theorem to be proved-must be true.

An indirect proof of validity for a given argument is constructed by assum- ing, as an additional premiss, the negation of its conclusion, and then deriving an explicit contradiction from the augmented set of premisses. Thus an indirect proof of validity for the argument

A⊃ (B-C)

(BvD) ⊃E

DvA

∴E

may be set down as follows:

1. A⊃D (B.C)

2. (BvD) ⊃E

3. DvA /∴E

4. ~E I.P. (Indirect Proof)

5.~(BvD) 2, 4, M.T.

6. ~B.~D 5, De M.

7. ~D.~B 6, Com.

8. ~D 7, Simp.

9. A 3, 8, D.S.

10. B.C 1, 9, M.P.

11. B 10, Simp.

12. ~B 6, Simp.

13. B.~B, 11, 12, Conj.

Unit V

Shorter Truth Table Method for proving Invalidity
Preliminary Set theory

1. What do you mean by providing invalidity with the help of shorter method of truth table.

Ans: Construction of truth-table is the foundation of propositional calculus. It is not possible to prove invalidity without its help. The method is quite simple. Irrespective of the number of propositions, the entire operation can be completed in one straight line. We devise a single, row represented by a straight line. All variables, their negations and all compound propositions are arranged horizontally above the straight line. The truth-value is entered exactly below the respective proposition. It is not necessary that the truth-value of every variable has to be entered. It depends upon the situation. The straight line separates the variables and the respective truthvalues. Calculation of truth-value of propositions follows the elementary principles of propositional calculus. Since we have to prove that the argument is invalid, '0' is the truth-value to be assigned necessarily to the conclusion. This is the first step to be followed. If the conclusion is a compound proposition, then the sentential connective must be assigned the value 'F'. Suppose that in the conclusion there are multiple sentential connectives. Then the sentential connective which has maximum range must be assigned the designate value. If implication links the antecedent and the consequent, then implication will have maximum range. Therefore in such case the implication must be assigned this value. In all other cases a little examination is adequate to identify the connective with maximum range. In the next step, all premises must be assigned the truth-value 'T' only. Again, if any premise is a compound proposition then same method which is applicable to compound conclusion also applies to the premise or premises. A word of caution is required. It may not be possible to assign the required truth-value always in the very first attempt. We may have to take to trial-and-error method at times. This is more so when the premises are quite lengthy with multiple sentential connectives. It is also possible that more than one combination of truth-values for the variables of the premises may yield the desired result. Suppose that it is impossible to assign the truth-values in any the manner described above. Then the argument must be regarded as valid. Therefore this method can be used to prove validity also. However, we are concerned with proving invalidity at present. At this stage one clarification is required. Unlike validity, invalidity is not governed by any rules. Of course, it is more than obvious that errors do not have any rules, which govern. On the other hand, only violation is possible. Hence the method of proving invalidity is different. The principle of inference dictates that a true premise and a false conclusion together result in invalidity. This is the reason why in order to determine invalidity we should assign truth-values in such a way that the premise or premises are true and the conclusion is false. If we succeed in doing so then the argument is invalid. This method is so simple that the test can be completed in one line (or two lines depending upon the number of variables and constants) as it happens in the case of truth-table. So far we concentrated on the theoretical aspect. We shall apply now this method to an argument. The conclusion finds place at the end of the line always.

Example:

Consider the invalid argument

If the Senator votes against this bill, then he is opposed to more severe penalties against tax evaders.

If the Senator is a tax evader himself, then he is opposed to more severe penalties against tax evaders.

Therefore, if the Senator votes against this bill, he is a tax evader himself.

which may be symbolized as

$V \supset O$

$H \supset O$

$\therefore V \supset H$

Instead of constructing a truth table for the specific form of this argument, we can prove its invalidity by making an assignment of truth values to the component simple statements V, O, and H, which will make the premisses true and the conclusion false. The conclusion is made false by assigning T to V and F to H; and both premísses are made true by assigning T to O This method of proving invalidity is closely related to the truth table method. In effect, making the indicated truth value assignment amounts to describing one row of the relevant truth table-a row that suffices to establish the invalidity of the argument being tested. The relationship appears more clearly, perhaps, when the truth value assignments are written out horizontally:

V OH $V \supset O$ $H \supset O$ $V \supset H$

T T F.   T   T   F

This new method of proving invalidity is shorter than writing out a complete truth table. The amount of time and work saved is proportionally greater for more complicated arguments. In proving the invalidity of more extended arguments, a certain amount of trial and error may be needed to discover a truth value assignment that works. But even so, this method is quicker and easier than writing out the entire truth table. It is obvious that the present method will suffice to prove the invalidity of any argument that can be shown to be invalid by a truth table.

2. What is predicate logic?

Ans: Predicate logic, is a branch of logic, which is concerned with predicates or with predication of properties, and also with things or objects to which the predicates may be ascribed. This statement, in general, and the words in italics, in particular, must act as springboard for our further study and its significance becomes evident shortly. Quantification logic has its roots in 'Set Theory'.

3. What is Set theory?

Ans: Set theory itself is of recent origin. This theory took its birth in the nineteenth century in the works Georg Cantor. However, other mathematicians, most notably, Boole, Venn and de Morgan meant it though they did not develop the theory. Nor did they use the term in the sense in which Cantor used. Such things are not uncommon in the development of science. For example, Michael Faraday did not use the word field in his work on physics though he meant it. This piece of fact from the History of Mathematics must be borne in mind in order to pay what is due to Aristotle. Prior to this, we must understand that the important concepts developed by Cantor viz. subset, proper subset, the difference between the two, null set (empty set), denumerable and equivalence are some of the key concepts.

4. Where is the error in Aristotle's theory?

Ans: As a matter of Aristotle did not err. That is the reason why the word defect is not the right word to be used while assessing Aristotelian system. Instead, limitation is the apt word to be used in our analysis of Aristotelian system. Aristotle had an idea of class at elementary level. He evolved the concepts of class inclusion (total or partial) and class exclusion (total or partial), but could not proceed further. This explains the limits of his analysis of categorical proposition based on quality and quantity of proposition and the outcome of his analysis. Since Set theory in the sense in which Cantor developed was unknown during Aristotle's age, it, surely, would be anachronistic to criticize Aristotle for his limited perspective of predicate logic. Therefore let us first identify the loose ends in Aristotelian system. This will help us to understand the significance of 'Quantification Logic' in particular and modern logic in general. Aristotle did not differentiate between universal proposition and singular proposition. A proposition is singular when the subject is a proper name. In this respect, singular proposition differs from particular proposition, though later we understand that both are existential propositions. In his analysis these two are, more or less, the same. An understanding of subtle difference and its consequences is quite illuminating. Any universal proposition of the form ‘All S are P’ or ‘No S are P’ reveals that S and P are merely class-names.

5. What is called Set?

Ans: If the concept of denotation is closely examined, then it becomes clear that all class-indicators include or exclude a certain number of elements known as members of a particular class, otherwise called sets. Therefore every set represented by a term in the proposition is very much similar to denumerable set which is a set of positive integers. A set is said to be denumerable when it is a set of positive integers because only then members are countable. If members are countable, then denotation makes sense, not otherwise. Similarly, the concept of intension reveals that to be a well-defined member the member must possess a definite set of properties without which it ceases to be a member of that particular set.

Against this background, we should try to know what the difference or differences between universal and singular on the one hand and particular and singular propositions on the other signify.

First let us consider universal and singular propositions. The proposition ‘All men are mortal’ has both contrary and contradictory relations. However, the proposition ‘Socrates is mortal’ has only contradictory relation, but not contrary. It may be necessary to point out that, though it amounts to repetition, two propositions are contraries only when two conditions are satisfied; when p is true, q is false and when p is false q is doubtful. On the other hand, contradiction arises when p is true, q is false and when p is false q is true and vice versa.

Suppose that the second proposition, ‘Socrates is mortal’ is negated. We get ‘Socrates is not mortal’. When the first statement is true, the second statement is false. Though the first condition is satisfied, the second condition is not satisfied because when the first statement is false, the second statement is not doubtful, but turns out to be true. If logical relations matter, then the distinction between universal and singular proposition also ought to matter. This is a point which Aristotle failed to notice. Further, both particular and singular are existential propositions which make matters still worse. Like universal propositions, particular propositions also have two distinct relations which distinguish them from singular propositions. Instead of contrary, sub-contrary explains one type of relation between two particular propositions. If ‘some men are mortal’ is true, then ‘some men are not mortal’ is doubtful and if ‘some men are mortal’ is false, then ‘some men are not mortal’ is true. Of course, contradiction explains the relation between universal and particular. Here is the difference. Though both particular and singular propositions are existential, sub-contrary relation is not common to both. This means that universal and particular propositions, on the one hand, and particular and singular, on the other, deserve to be classified separately. They are called general propositions distinct from singular propositions because the subject of such propositions is a general term. A term which refers to an indefinite number of things is a general term which is called common noun in grammar. What we call quantifiers are applicable to general propositions, but not to singular propositions. This is one difference. Second difference is crucial. In this context, the emphasis is on the word existence.

6. If a certain proposition is characterized as existential, how do we understand such characterization?

Ans: When we discussed Venn's diagram in connection with the distribution of terms, we learnt that universal propositions do not carry existential import whereas particular propositions carry existential import. The statement 'All men are mortal' does not affirm the existence of men whereas 'Some men are….' affirm the existence of men irrespective of the quality of proposition. Same is the case with 'No men are…' No assertion is made about the existence of men. Existence presupposes the presence of members in a given class. If existence makes sense, then in negative sense nonexistence also must make some sense. Suppose that a set does not contain a single member. Then what is its status? Till nineteenth century this question did not occur to anyone. In other words, the concept of null set paved the way for further progress in Aristotelian logic.

7. Discuss nul set .

Ans: The concept of null set plays crucial role in distinguishing Aristotelian system from modern logic. Let us recall the very first statement of introduction; 'Predicate logic, is a branch of logic, which is concerned with predicates or with predication of properties, and also with things or objects to which the predicates may be ascribed'. In the strict sense of the term, predicate may be ascribed to only things or individuals actually existing. Otherwise, the commonplace difference between fact and fiction will be completely obliterated. Therefore in a restricted sense existential propositions make matters of fact relevant. When matters of fact become relevant, purely formal character of formal logic makes room for the relevance of content to a certain extent. But generalization, which is a characteristic of deductive inference, does not lose its significance. The only requirement is that the content of the argument must be factual, but not fictitious. One fundamental relation between propositions with which we are concerned, presently, is contradiction. The law of contradiction holds good when terms include members as a matter of fact. However, the situation is different when the terms represent null sets. Consider this proposition.

1. All female philosophers of Karnataka are the residents of New York.

This sentence is obviously false. Therefore according to the law of its contradiction, the statement mentioned below must be true.

2. Some female philosophers of Karnataka are not the residents of New York.

Statements 1 and 2 are supposed to be contradictories. The second statement ought to be true according to the law of contradiction since the first statement is false. But, in reality, this statement is also false. But two contradictories cannot be false. This problem arises because we are dealing with nonexistent members. Therefore in the strict sense of the word second statement also, like universal statement, does not carry existential import. Within the framework of traditional logic this problem remains unnoticed because there was no concept of set at all-whether null set or non-null set. Modern logic corrected this mistake by making null set a distinct entity. The underlying principle is that all existential propositions should include only non-null sets. This stipulation marks one difference between traditional and modern systems. Equivalence is second major factor. Consider these propositions.

3. All triangles are plane figures.

4. All equilateral triangles are equiangular triangles.

Statements 3 and 4 assume the forms as follows.

3a. If any figure is a triangle, then it is a plane figure.

4a. A figure is equilateral triangle if and only if it is equiangular.

Let us symbolize these statements.

3a ≡ F => P

3b ≡ F <=> P

Again, traditional logic did not distinguish these propositions. The difference between 3a and 3b becomes clear only within the framework of modern logic. This is another important progress made by modern logic over traditional logic. Such differences matter in quantification logic. This aspect has something to do with the difference between subset and proper subset. Proposition 3 discloses that the set of triangles is a proper subset of the set of plane figures.

However, the set of equilateral triangles is equivalent to the set of equiangular triangles. This explains why the sentential connectives differ from 3a to 3b.

Traditional logic made one type of distinction among propositions; conditional and unconditional and within the latter four kinds of propositions. Modern logic not only discovered a new aspect in conditional proposition but also it added new kinds of propositions which were not considered by traditional logic. Hence it could evolve new set of rules. But these rules had limitations. In the absence of further augmentation of new rules they could not be applied to arguments which consisted of singular propositions. So the search for relevant rules did not stop.

8. What is the difference between propositional logic and predicate logic?

Ans: This difference lies in dealing with the internal structure of simple as well as compound propositions. Therefore predicate logic includes rules hitherto used and also a new set of rules.

Predicate logic is concerned with the internal structure of propositions. It is not the case with propositional logic. In strict technical terms, this is also known as Quantification Theory or the

Predicate Calculus. It has its own syntax, which helps us to devise statements, which are considered well-formed statements.