

**2023/FYUG/ODD/SEM/
MATDSM-101T/142**

**FYUG Odd Semester Exam., 2023
(Held in 2024)**

MATHEMATICS

(1st Semester)

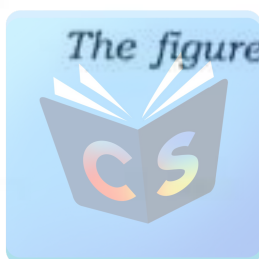
Course No. : MATDSM-101T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours



*The figures in the margin indicate full marks
for the questions*

CAREER STUDY

The Learning App

SECTION—A

Answer ten questions, selecting any *two* from each

Unit :

2×10=20

UNIT—I

- 1. State Cauchy's criterion for the existence of limit of a function.**
- 2. Show that $\lim_{x \rightarrow 0} \cos\left(\frac{1}{x}\right)$ does not exist.**

(2)

3. Show that if a function $f(x)$ is differentiable at a point $x = a$, then it is also continuous at $x = a$.

UNIT—II

4. Give the geometrical meaning of Rolle's theorem.
5. In the mean value theorem

$$f(x+h) = f(x) + hf'(x + \theta h)$$

If $f(x) = Ax^2 + Bx + C$, where $A \neq 0$, show that

$$\theta = \frac{1}{2}$$

6. Evaluate :



$$\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$$

UNIT—III

7. Define homogeneous function of degree n of two variables. Is the function $(\sqrt{x} + \sqrt{y} + \sqrt{z})$ is homogeneous? If so find its degree.
8. If $f(x, y) = e^{x^2 + xy + y^2}$, find f_{xx} and f_{xy} .

(3)

9. If $u = f\left(\frac{y}{x}\right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$

UNIT—IV

10. If $f(x)$ is an even function, show that

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

11. If $f(x) = x - [x]$, then evaluate

$$\int_{-1}^1 f(x) dx$$

where $[x]$ is the integral part of x .

12. Evaluate :

$$\int_0^{\pi/2} \sin^8 x \cos^6 x dx$$

UNIT—V

13. Find the length of arc of the parabola $y^2 = 16x$ measured from vertex to an extremity of the latus rectum.

(4)

14. What do you mean by rectification of plane curve? Write the formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$.
15. Find the surface area of a solid generated by revolving the semicircular arc of radius c about the axis of x .

SECTION—B

Answer *five* questions, selecting *one* from each Unit :

10×5=50

UNIT—I

16. (a) Using ϵ - δ definition of limit, evaluate

$$\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x}\right)$$

5

- (b) Examine the differentiability of the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at $x = 0$.

5

17. (a) If $y = \sin(m \sin^{-1} x)$, then show that
 $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$
 Also find $y_n(0)$.

5

- (b) State and prove Leibnitz's theorem on successive differentiation.

1+4=5

UNIT—II

18. (a) State and prove Lagrange's mean value theorem.

5

- (b) Evaluate :

5

$$\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$$

19. (a) Show that the maximum value of $x^{1/x}$ is $e^{1/e}$.

5

- (b) Write the statement of Maclaurin's theorem. Also expand $\sin x$ using Maclaurin's infinite expansion.

1+4=5

UNIT—III

20. (a) State and prove Euler's theorem on homogeneous function of degree n in two variables x and y .

5

(6)

- (b) Find the tangent and normal to the curve

$$y(x-2)(x-3) - x + 7 = 0$$

at the point where it cuts the x -axis. 5

21. (a) If

$$V = \log \left(\frac{x^3 + y^3}{x^2 + y^2} \right)$$

show that

$$x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = 1 \quad 5$$

- (b) Show that at any point on the curve

$$x^{m+n} = k^{m-n} y^{2n}$$

the m th power of the subtangent varies as the n th power of the subnormal. 5

UNIT—IV

22. (a) Obtain the reduction formula for

$$\int \sin^m x \cos^n x \, dx$$

where m, n are positive integers > 1 . 5

- (b) Evaluate : 5

$$\int_0^{\pi/2} \left(\frac{1}{1 + \cot x} \right) dx$$

(7)

23. (a) Obtain the reduction formula for

$$\int \sec^n x \, dx$$

n being a positive integer greater than 1. 5

- (b) Prove that

$$\int_0^{\pi/2} \log(\sin x) \, dx = -\frac{\pi}{2} \log 2 \quad 5$$

UNIT—V

24. (a) Find the area of the astroid

$$x^{2/3} + y^{2/3} = a^{2/3} \quad 5$$

- (b) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. 5

25. (a) Find the surface area of the solid generated by revolving the cycloid

$$x = a(\theta + \sin \theta), \quad y = a(1 + \cos \theta)$$

about its base. 5

- (b) The circle $x^2 + y^2 = a^2$ revolves around the axis. Find the surface area and the volume of the whole surface generated. 5
