

**2023/FYUG/ODD/SEM/  
MATDSC-102T/141**

**FYUG Odd Semester Exam., 2023  
( Held in 2024 )**

**MATHEMATICS  
( 1st Semester )**

**Course No. : MATDSC-102T**

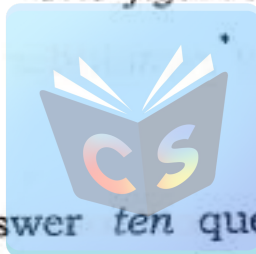
**( Differential Calculus )**

*Full Marks : 70*

*Pass Marks : 28*

*Time : 3 hours*

*The figures in the margin indicate full marks  
for the questions*



**CAREER STUDY**  
**SECTION—A**  
The Learning App

**Answer ten questions, selecting two from each**

**Unit :**

**2×10=20**

**UNIT—I**

1. Find the value of  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{\tan x} \right)$ .

2. Find the points of discontinuity of the  
function  $\frac{x^2 + 2x + 5}{x^2 - 8x + 12}$ .

( 2 )

3. Find the derivative of  $\frac{\sin x}{x}$ .

## UNIT—II

4. Evaluate  $\lim_{x \rightarrow 0} \frac{(e^x - 1)\tan^2 x}{x^2}$ .

5. Find  $y_n$  if  $y = \sin^3 x$ .

6. Find the range of values of  $x$  for the function

$$x^3 - 3x^2 - 24x + 30$$

## UNIT—III

7. Write down the Cauchy's form of remainder in Taylor's expansion.

8. Write down the geometrical interpretation of mean value theorem.

9. Is Rolle's theorem applicable for the function  $f(x) = \tan x$  in  $(0, \pi)$ ?

## UNIT—IV

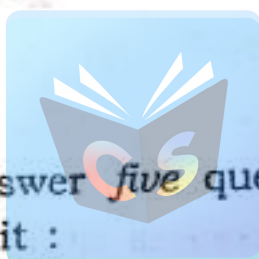
10. Write down the condition of orthogonality of the curves  $f(x, y) = 0$ ,  $\phi(x, y) = 0$ .

( 3 )

11. What is the geometrical meaning of  $\frac{dy}{dx}$ ?
12. Write down the formula for polar sub-tangent and polar subnormal of the curve  $r = f(\theta)$ .

## UNIT—V

13. What do you mean by point of inflection?
14. When is a function said to be a homogeneous function?
15. What do you mean by asymptote of a curve?



## SECTION—B

Answer five questions, selecting one from each

Unit :

10×5=50

## UNIT—I

16. (a) What do you mean by removable discontinuity? Show that a function which is continuous throughout a closed interval is bounded therein.  $2+4=6$
- (b) Show that  $\lim_{x \rightarrow 0} \cos \frac{1}{x}$  does not exist. 4

( 4 )

17. (a) Define continuity of a function. A function  $f(x)$  is defined as

$$\begin{aligned} f(x) &= 3 + 2x \quad \text{for } -\frac{3}{2} \leq x < 0 \\ &= 3 - 2x \quad \text{for } 0 \leq x < \frac{3}{2} \\ &= -3 - 2x \quad \text{for } x \geq \frac{3}{2} \end{aligned}$$

Show that  $f(x)$  is continuous at  $x=0$  and discontinuous at  $x=\frac{3}{2}$ . 2+4=6

- (b) Find the derivative of  $x^x$  from first principle. 4

### UNIT—II

18. (a) State L' Hospital theorem. Evaluate

$$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x} \quad \text{2+4=6}$$

- (b) If  $y = \frac{1}{x^2 + a^2}$ , then find  $y_n$ . 4

19. (a) If the area of a circle increases at a uniform rate, then show that the rate of increase of the perimeter varies inversely as the radius. 3

- (b) If  $y = \sin(m \sin^{-1} x)$ , then show that—

(i)  $(1 - x^2)y_2 - xy + m^2y = 0;$

(ii)  $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$

2+3=5

( 5 )

- (c) Find the  $n$ th derivative of  $\frac{1}{x^2 - a^2}$ . 2

### UNIT—III

20. (a) Prove that

$$\lim_{h \rightarrow 0} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$$

provided  $f''(x)$  is continuous. 4

- (b) Expand  $\log(1+x)$  in powers of  $x$  in infinite series stating each of the condition under which the expansion is valid. 4

- (c) State Rolle's theorem. 2

21. (a) If  $f(x) = x^2$ ,  $\phi(x) = x$ , then find a value of  $\xi$  in terms of  $a$  and  $b$  in Cauchy's mean value theorem. 3

- (b) If

$$f(x) = \begin{vmatrix} \sin x & \sin \alpha & \sin \beta \\ \cos x & \cos \alpha & \cos \beta \\ \tan x & \tan \alpha & \tan \beta \end{vmatrix}, \quad 0 < \alpha < \beta < \frac{\pi}{2}$$

then show that  $f'(\xi) = 0$ , where  $\alpha < \xi < \beta$ . 4

( 6 )

(c) In the mean value theorem

$$f(x+h) = f(x) + hf'(x+\theta h)$$

find  $\theta$ , where  $f(x) = \frac{1}{x}$ .

3

### UNIT—IV

22. (a) If  $x \cos \alpha + y \sin \alpha = P$  touch the curve  $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$ , then show that

$$(a \cos \alpha)^{\frac{m}{m-1}} + (b \sin \alpha)^{\frac{m}{m-1}} = P^{\frac{m}{m-1}} \quad 4$$

(b) Find the angle of intersection of the curves  $r = a \sin 2\theta$ ,  $r = a \cos 2\theta$ .

3

(c) Show that in any curve

$$\frac{\text{subnormal}}{\text{subtangent}} = \left( \frac{\text{length of normal}}{\text{length of tangent}} \right)^2 \quad 3$$

23. (a) Show that the curves  $r^n = a^n \cos n\theta$  and  $r^n = b^n \sin n\theta$  cut orthogonally.

3

(b) (i) Find  $\frac{dS}{d\theta}$  of the curve  $r = a(1 + \cos \theta)$ .

(ii) Find  $\frac{dS}{dr}$  of the curve  $r = a\theta$ . 2+2=4

(c) Find the polar subtangent of  $r = ae^{\theta \cot \alpha}$ .

3

## UNIT—V

24. (a) If  $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$ , then show that—

$$(i) \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = x\phi\left(\frac{y}{x}\right)$$

$$(ii) \quad x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0 \quad 2+4=6$$

(b) Find the asymptotes of the curve

$$2x(y-5)^2 = 3(y-2)(x-1)^2 \quad 4$$

25. (a) State and prove Euler's theorem on homogeneous function  $f(x, y)$ .  $1+3=4$

(b) If  $u = F(y-z, z-x, x-y)$ , then prove that



$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

3

(c) Find the asymptotes of the curve

$$x(x-y)^2 - 3(x^2 - y^2) + 8y = 0 \quad 3$$

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